Revisiting the *k*-Neighbors Connectivity Problem under Practical Indoor Scenarios

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Abstract. *A important goal for Topology Control Protocols is to determine the least amount of neighbors needed to build a fully connected network, aka the k-neighbors connectivity problem. In this work, we estimated the minimum k-neighbors value for a given Wireless Sensor Network (WSN) under practical office scenarios using parameterized simulations that took into account office' size and layout, path loss exponent, and multipath fading. Our findings argue that the results of the k-neighbors connectivity problem based solely on flat scenarios or performed analytically can be misleading for the deployment of WSNs in indoor scenarios with obstacles, and suggest that the interplay of office size and layout with path loss exponent and multipath fading should be strongly considered on the development of future WSNs.*

1. Introduction

The technical challenges posed by Wireless Sensor Networks (WSN) have attracted the attention of networking researchers in the last years, especially because WSNs established new information collection and broadcast models, with huge potential for new applications such as environmental monitoring, "intelligent" buildings, to name a few.

On the other hand, a major constraint of WSNs is the limited battery life of the sensor nodes, which affects directly the network lifetime. Therefore, it is necessary that each node employs an effective Transmission Power Control (TPC) to maintain a given connectivity degree [Xue and Kumar 2004]. However, minimizing the energy consumption and maintaining the connectivity of a WSN are conflicting goals that influence other aspects of operation of the network nodes, such as the quality of the signal received, the range of transmission available, and the magnitude of interference among neighboring nodes, among others.

An example of technique to deal with TPC setup is Topology Control (TC). In theory, TC protocols (TCPs) need to be fully distributed and use only local information for decision-making in reducing traffic overhead. Thus, such a Distributed TCP (DTCP) should have the following characteristics [Santi 2005]: to be fully distributed and asynchronous; to be localized; to generate a topology that preserves the original network connectivity and rely, if possible, on bidirectional links; to generate a topology with small physical degree; and to count on 'low-quality' information. Although DTCPs can follow several approaches, including Location-based TC, Direction-based TC, and Neighbor-based TC, they eventually depend on the nodes' ability: (i) to determine the number and identity of neighbors within the maximum transmitting range; and (ii) to build an ordering of the neighboring set, e.g., based on link quality. In summary, DTCPs have to minimize the k-neighbors variable while keeping the overall connectivity.

To address such a problem, we used *Zerkalo*, our propagation analysis tool introduced in a previous work [Kostin and de Amorim 2007], to simulate the deployment of a WSN in practical office scenarios. By using *Zerkalo*, we could determine the minimum suitable k, for each scenario, that allowed the overall connectivity to be achieved considering the electromagnetic effects caused by propagation barriers and multipath fading. Although analytical models provide a probabilistic description, multipath fading is a deterministic phenomenon. Moreover, in the case of static nodes, fading is time-invariant.

In overall, the main contributions of this work are: (a) to find the number k for several parameterized scenarios as a function of N (the amount of WSN nodes); (b) to analyze the impact of office' size and layout, path loss exponent, and the impact of electromagnetic phenomena on the k-neighbors problem within indoor parameterized scenarios.

The remainder of the paper is organized as follows. Section 2 briefly presents related work. In Section 3, we introduce our simulation tool and the physical parameters assumed in the simulations. Section 4 presents the experimental evaluation we conducted, including a description of the simulated indoor scenarios, the results we obtained, and our main findings that improved on previous results in the literature. Finally, our conclusion and ongoing works are presented in Section 5.

2. Related Work

Topology Control (TC) is the art of coordinating nodes' decisions in order to generate a communication network with the desired properties considering either their transmitting ranges [Santi 2005], [Karl and Willig 2005] or their hierarchical topology organization [Bao and Garcia-Luna-Aceves 2003].

Given that nodes form transmission links by choosing the power level at which they will transmit, then a significant question posed to TC is to determine how many neighbors should each node be connected to, so that the overall network connectivity is achieved, aka the *k-neighbors connectivity problem* [Santi 2005]. This problem is formally defined as follows:

"Given a set N of nodes, what is the minimum value of k such that the k-neighbors graph G_k built on N is strongly connected?"

In practice, the higher the k, the better is the network connectivity. On the other hand, a small value of k is desirable for spatial reuse. Thus, the optimal choice of k is the minimum value of k such that the corresponding G_k graph is connected.

Works in the 1970s and 1980s suggested that the "*magic number*" of nearest neighbors should be either *six* [Kleinrock and Silvester 1978] or *eight* [Hou and Li 1986].

However, it has been recognized that to theoretically determine k is a hard problem (no "*magic number*" exists), which has been partially solved only recently [Xue and Kumar 2004].

Assuming N nodes placed uniformly at random, [Xue and Kumar 2004] showed that k is given by $k = c.logN$ for some constant c, with $0.074 < c \leq 5.1774 + \epsilon$ where ϵ is an arbitrary small positive value.

The k-neighbors problem was also investigated in [Blough et al. 2003], which simulated flat scenarios in details, obtaining values that were compatible with [Xue and Kumar 2004]. In particular, Table 1 summarizes their reported results for the minimum value of k that guaranteed a connected topology with probability of at least 0.95.

[Bettstetter and Hartmann 2005] analyzed the connectivity of multihop radio networks in a log-normal shadow fading environment, assuming that the nodes had equal transmission capabilities and were randomly distributed according to a homogeneous Poisson process, thus providing a tight lower bound for the minimum node density that is necessary to obtain an almost connected subnetwork on a bounded area of a given size.

Usually, the k-neighbors problem is analyzed in an analytical way. [Miorandi and Altman 2005] presented an analytical procedure for computing the node isolation's probability in an ad hoc network in the presence of channel randomness, with applications to shadowing and fading phenomena. Those results were used to obtain an estimate of connectivity features for very dense networks, but the authors did not evaluate their estimation in a deterministic way.

[Mullen and Huang 2005] examined several Mobile Ad Hoc Networks (MANETs) behaviors and analyzed their root causes using a stochastic model of received power, specifically they examined the difficulties in estimating the mean signal-to-noise ratio in a MANET through mathematical analysis.

[Puccinelli and Haenggi 2006] illustrated the spatial nature of multipath fading with experimental evidence obtained using lower-end sensing node hardware, showing also the limitations of a supposed immunity of wideband radios to multipath fading in indoor deployments.

A practical approach to solving the k-neighbors problem is the use of Neighbor-based DTCPs. These protocols are based on the node's ability to determine the number and identity of neighbors within their range and to build an order on this neighboring set (based on distance such as KNEIGH [Blough et al. 2003] or on link quality, e. g., KNEIGHLEV [Blough et al. 2006] and S-XTC [Dyer et al. 2007]). In a certain sense, this is the minimum amount of information needed by the nodes to build the network topology, allowing a network construction with simpler hardware.

The motivation for this work is based on the observation that we have not found any work in the literature that analyzed the k-neighbors connectivity problem considering propagation barriers and multipath fading in a deterministic way.

3. Simulation Tool

In this section, we briefly explain Site Specific Propagation (SISP) Models and our tool, *Zerkalo*.

Table 1. Critical Neighbor Number for Different Values of N

N	k(P>.95)	k(P=1)	N	k(P>.95)	k(P=1)
10	6	6	100	7	9
20	7	8	250	7	9
25	7	8	500	6	9
50	7	9	1000	6	10

3.1. Site Specific Propagation Model

The simplest software approach to radio-wave propagation modeling at high frequencies (VHF to SHF) is semi-empirical, such as the well-known exponential path-loss model. Radiowave propagation models using detailed terrain databases are commonly referred as SISP models [Seidel and Rappaport 1991]. Small scenarios (usually indoors) may benefit from more complex and accurate approaches such as ray-tracing modeling. In this technique, the main propagation paths (rays) are deterministically found based on the common electromagnetic phenomena of reflection, refraction, and scattering, including diffraction. Ray-tracing is usually carried out two-fold, using greedy methods or image theory [Sarkar et al. 2003]. With the ever-growing available numerical capacity of computers, ray-tracing models have increasingly become more attractive as propagation prediction tools. Some networking researchers even expect that deterministic modeling may prevail in a near future, as the approach of choice for propagation prediction, even outdoors [Rappaport 2002].

3.2. Zerkalo

In a previous work [Kostin and de Amorim 2007], we developed a SISP tool called *Zerkalo* (*mirror* in Russian) based on ray-tracing (images method) [Sarkar et al. 2003] that simulates the electromagnetic propagation in a given scenario using discrete event simulation techniques. Besides free-space propagation, *Zerkalo* also simulates the electromagnetic phenomena of reflection and refraction by computing the multipath interference due to reflections up to a desired order. *Zerkalo*'s algorithm complexity is O(n^r), where r is the reflection order and n is the number of obstacles. This complexity value is usually expected for ray-tracing based algorithms, including Zerkalo itself.

In the design of *Zerkalo*, we assumed the so-called *narrowband* hypothesis that considers that the transmitted signal's spectral content is narrow enough around the carrier (dozens or hundreds of KHz depending on the conditions) so that the technique of fading can be considered flat [Rappaport 2002]. The regions most affected by this kind of fading are those closer to walls, specially the ones near to the corners [Puccinelli and Haenggi 2006].

We have also assumed the following test parameters: 0.122 m wavelength, half wave dipole antennas (1.64 dBi) for transmission and reception [Rappaport 2002], the transmission power varies from -20 dBm to 20 dBm, CSMA/CA-based MAC, receiver sensitivity is -70 dBm, carrier sense threshold is -85 dBm, the capture threshold is 10 dB, and considering up to second order reflections. We have modeled the error in the Received Strength Signal Intensity (RSSI) as 10% of receive power around the correct value. In all scenarios, the antennas' heights were half way between floor and roof, such that the major propagation effects were concentrated on the horizontal plane comprising all antennas, simplifying the propagation problem to a 2D analysis.

For the simulations considering multipath, we assumed a multipath fading threshold value of half the power received in the main propagation path, which usually is the direct path. Specifically, whenever the (complex) sum of all multipath phasors is up to such threshold (equivalent to a 3 dB difference, at least), the signal will be codified, otherwise it will not.

The Neighbor-Based DTCP protocol chosen as reference is the S-XTC [Dyer et al. 2007], which is suitable for static networks in most cases and there are practical implementations of it. Therefore, is based on the concept of 'link quality'. In our case, the 'link quality' corresponded to a combination of RSSI and multipath fading, as described earlier.

We consider S-XTC [Dyer et al. 2007], as *state-of-the-art* of neighbor-based DTCP because in contrast to the others similar protocols S-XTC add to the robustness and resilience against fluctuation in the RSSI values. While guaranteeing connectivity and a bounded node degree, network topologies are able to adapt to gradual changes in the network and its environment.

4. Experimental Evaluation

In this section, we present the test scenarios, compute the k variable according to N for each environment, and carried out an analysis of the effects in isolation and in combination of office' size and layout, the path loss exponent, and multipath fading on the results.

4.1. Scenarios

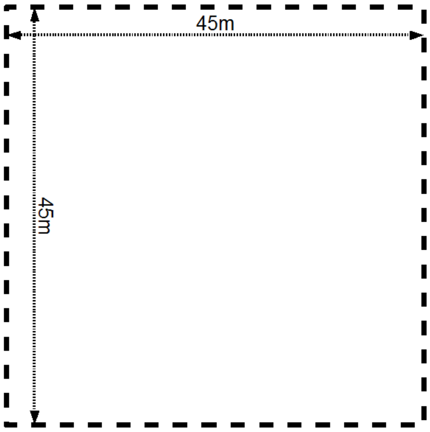
As test scenarios we included four indoors and the flat one, as shown in Figure 1. Two of the test scenarios were standard for analyzing the k-neighbors problem and the remaining ones were based on practical scenarios extracted from [Kubisch et al. 2003] and [Monteiro et al. 2007].

4.1.1. Flat Scenario

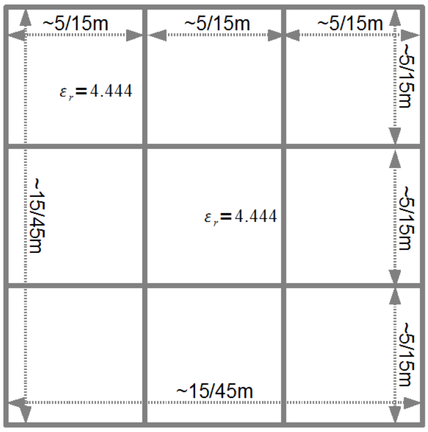
We chosed a flat scenario of 45×45 m^2, shown in Figure 1(a) in order to provide results for comparison with those of scenarios with obstacles, allowing us to examine the differences that obstacles might cause to the k-neighbors problem.

4.1.2. Tic-Tac-Toe Scenario

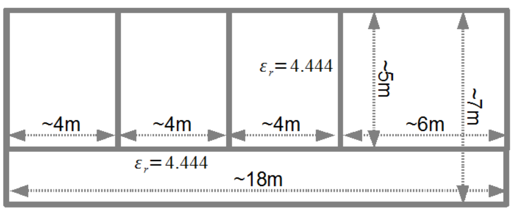
The standard test scenario, as shown in Figure 1(b), was composed of nine rooms, similarly arranged as in the *Tic-Tac-Toe* game. The rooms are apart by 15 cm wide brick walls with relative permittivity (ε_r) equal to 4.444 [Rappaport 2002]. We used two different Tic-Tac-Toe scenarios in order to compare the effects of office size on k. The first one, called normal Tic-Tac-Toe (TTT), where each room was 15×15 m^2. The second one, called small Tic-Tac-Toe (sTTT), used 5×5 m^2 rooms.



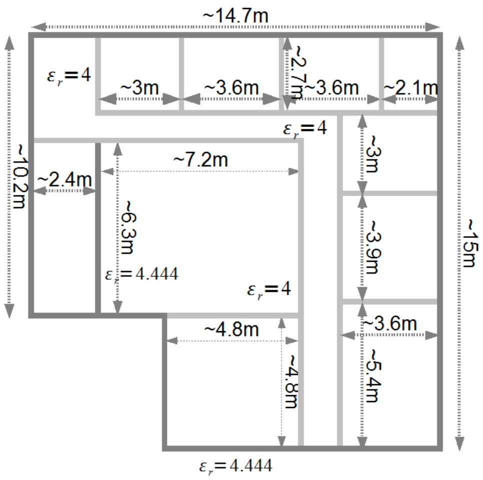
(a) Plain



(b) Small and normal TTT



(c) Kubisch



(d) PCL

Figure 1. Test Scenarios

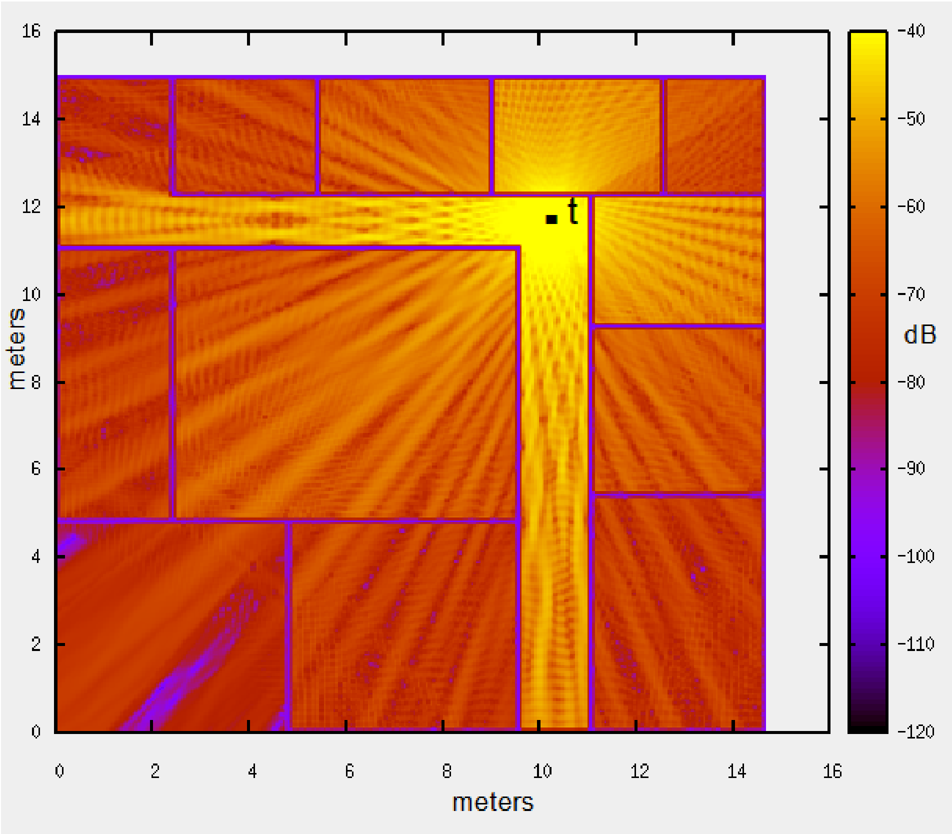


Figure 2. Behavior of Path Loss in PCL

4.1.3. Kubisch Scenario

Kubisch scenario, as shown in Figure 1(c), was copied from [Kubisch et al. 2003]. The physical layout (18×7 m^2) consisted of four rooms connected by a hallway as can be seen in the figure. In [Kubisch et al. 2003], the walls were assumed to be infinitesimally thin, so that they were no obstacle to radio communication. Assuming a practical scenario, we considered that the rooms were apart by brick walls with $\varepsilon_r = 4.444$ [Rappaport 2002].

4.1.4. Parallel Computing Laboratory (PCL)

Parallel Computing Laboratory (PCL) [Monteiro et al. 2007] is a typical office, shown in Figure 1(d) (14.7×15 m^2), divided by brick ($\varepsilon_r = 4.444$), standard wood ($\varepsilon_r = 4.000$) and glass walls($\varepsilon_r = 4.000$). Figure 2 illustrates how our SiSP tool computes path loss, considering multipath fading.

4.2. Results

To solve the k-neighbors connectivity problem for the test scenarios, we conducted 200 random simulations for each pair $k \times N$, according a random uniform distribution. The results are shown in Figure 3 for all test scenarios, each of which with and without multipath effects (except the flat scenario which has no multipath). The 95% and 99% curves correspond to the average value computed over the values collected when the connected

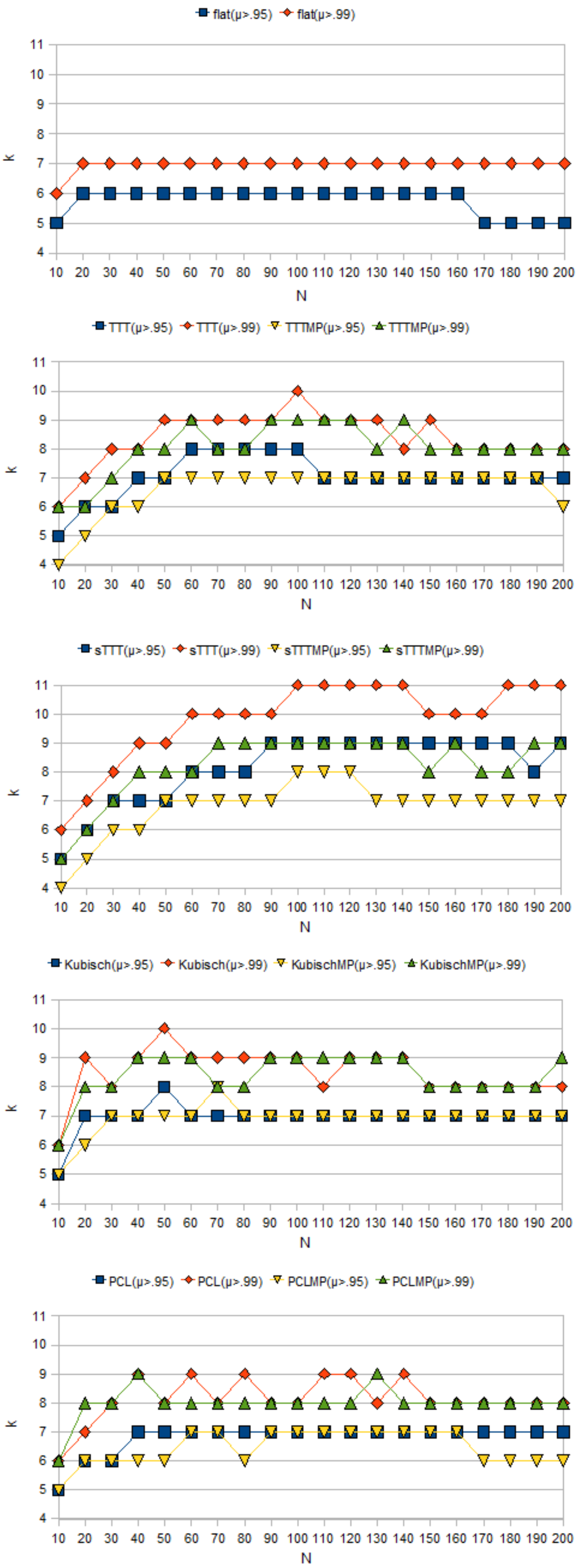


Figure 3. $k \times N$ Relationship for All Scenarios

Table 2. Path Loss Exponent (PLE) of Evaluation Scenarios

SCENARIO	PLE
Flat	2.00
Tic-Tac-Toe (TTT)	3.68
small *Tic-Tac-Toe* (sTTT)	3.86
Kubisch	3.08
PCL	3.69

nodes were greater than 95% and 99%, respectively, according to the parameters of the previous section.

We notice a small difference between the data of Table 1 and corresponding graphics of the flat scenario. The reason was the long simulation time required for scenarios with obstacles, which was alleviated by using the average instead of probability, yet obtaining equivalent results. Figure 3 confirmed the intuitive result that the obstacles should increase the k value. In all test scenarios, we obtained larger k values than those of flat areas. This result alone motivated as to study in depth the k-neighbors connectivity problem in a parameterized office scenario.

4.3. Analysis of Results

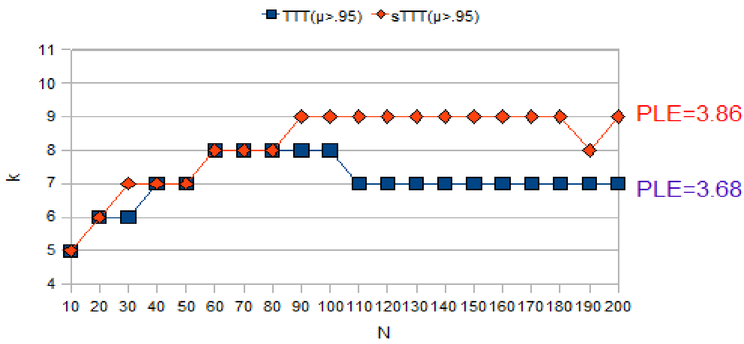
In this section, we will perform a detailed analysis of the results presented in the previous section. We decomposed the results of Figure 3 and used them to plot the graphics shown in Figure 4. With this figure, we will analyze the impact of the office' size and layout, and the multipath fading effect on the k-neighbors problem. The PLE of each scenario is shown in Table 2. It should be notice that given the improvements in hardware, wireless devices are becoming more resilient to multipath fading, though it still affects significantly current devices, as our results will point out shortly.

4.3.1. Office Size

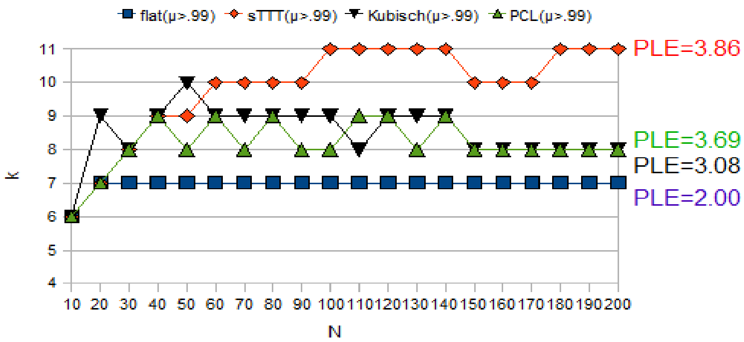
According to Figure 4(a), we note that in scenarios with the same layout and different sizes, the behavior of the graphics differs. Larger scenarios need smaller k values in order to keep the overall connectivity. In our case, the TTT scenario has a smaller PLE (3.68) than sTTT (3.86). This result could induce us to infer that scenarios with smaller PLEs will allow smaller k values, but the results presented next will show that it does not always occur. We also made tests changing the office wall's material, and, as we suspected, k grew as ε_r increased.

4.3.2. Office Layout and Path Loss Exponent

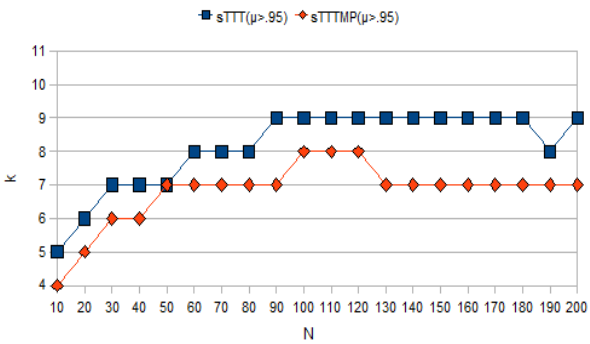
The office layout produced important results, as we can see in Figure 4(b). Although Kubisch and PCL PLEs were 3.08 and 3.69, respectively, in both cases the k values were almost the same. Yet, if we computed an weighted average ($\sum k_i.N_i/\sum N_i$), we noticed that PCL results were a bit smaller than those of Kubisch. Therefore, we can not make inferences on k based solely on PLE, the main parameter in several analytical models.



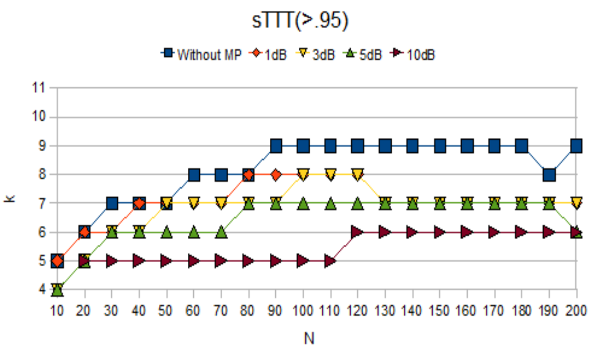
(a) Dimension Effect



(b) Layout Effect



(c) Multipath Fading Effect



(d) Multipath Fading Threshold Effect

Figure 4. Effect on Results

Unfortunately, this is also the case of generic simulators such as the NS-2, which does not consider scenario' specific conditions, and only allows the user to adjust the PLE, thus weakening the study of the K-neighbors connectivity problem under practical indoor scenarios.

4.3.3. Multipath Fading Effect

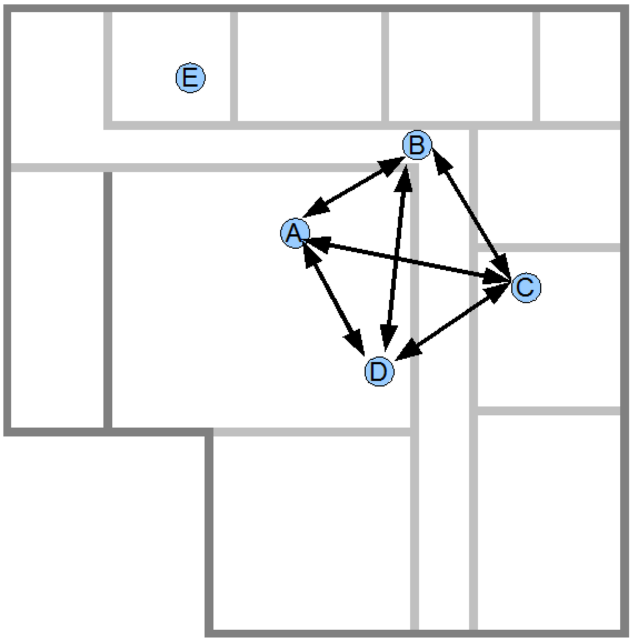
The multipath fading effect presents two behaviors in WSNs. When we have few sensor nodes, that effect has a negative impact, forcing the nodes to increase the k value. However, when N increases, the difference between the two approaches (whether considering multipath or not) favors the scenario that considers the effect. This occurs because the multipath fading effect obstructs links between nodes that are relatively close, demanding the sensor to apply a higher transmission power avoiding clustering. The result is that the overall network becomes more connected, although spending more energy. The beneficial effect of multipath fading is very clear in the small Tic-Tac-Toe scenario, as we can see in Figure 4(c), though a bit less in PCL, and slightly in Kubisch.

WSN nodes could have multipath mitigation techniques embedded in their circuits. The threshold will depend on the quality of such equipment, and as a result, the value of *k* will also depend on the equipment quality. Tests with other thresholds indicated that increasing the threshold leads to smaller *k* values (see Figure 4(d)). However, for WSN with few nodes, excessive high thresholds may prevent the network to achieve the connectivity goal.

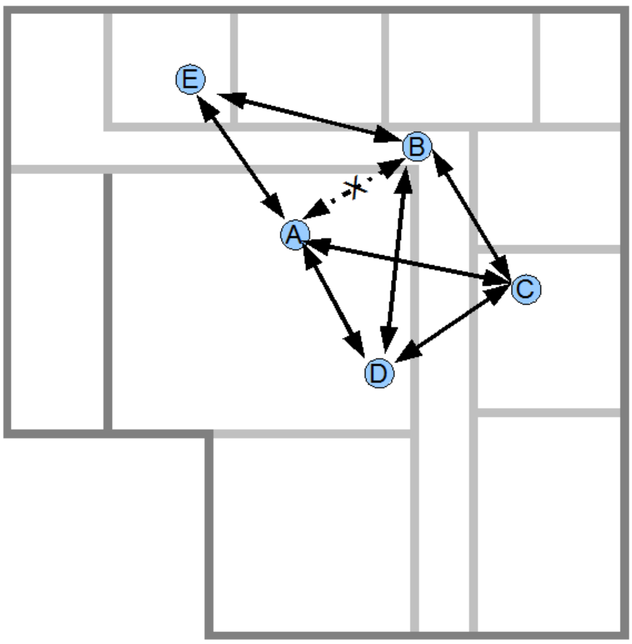
A possible explanation for these results can be understood with the help of Figure 5, which illustrates a hypothetical scenario with five nodes and k = 3, where four nodes (A, B, C and D) are sharing a network and node E is outside of it. According to the arrangement of Figure 5(a), node E will not take part of the overall network. On the other hand, it will take part of the network if k is set to 4, instead of 3.

However, if the multipath effect is considered, the link between A and B is subject to eventual deep fades, which may lead to high Bit Error Rates (BER) and even to link outage. Since the narrowband hypothesis has been assumed, frequency selective fading is less frequent, but still likely to happen. Under such particular situation, the link performance is severely degraded due to intersymbol interference (ISI), which can not be mitigated by simply increasing the transmission power level. Anyway, since nodes A and B need to be connected to three peers, they will increase their power until node E is contacted, thus forming new links (though spending more energy). In this example where multipath fading has been considered, k=3 was enough to keep the whole network connected.

Although the results discussed here were based on a few samples of the WSN behavior at indoor environments, they point out that in some realistic cases, given to the physical effects caused by obstacles, the achieved connectivity performance may contradict the expectations, as illustrated above. Moreover, it is worth noticing that deterministic SISP tools such as *Zerkalo* may be applied to any scenario that can be modeled in details both geometrically and electrically. As stated before, some networking researchers even expect that deterministic modeling may prevail in a very near future, as the preferred



(a) Example of Connectivity Without Considering Multipath Fading



(b) Example of Connectivity Considering Multipath Fading

Figure 5. Multipath Fading Affecting k

approach for propagation prediction, even outdoors [Rappaport 2002].

5. Conclusion

This work presented an in-depth deterministic analysis on the minimum k-neighbors value for a given WSN to build a fully connected network under practical indoor scenarios. We computed k for four parameterized office scenarios and a flat one, and discussed the effects of office' size and layout, path loss exponent, and multipath fading on the k-neighbors problem.

Our results revealed that the office walls increased the k values and that such increments were due to specific combinations of PLE, and office' size and layout. For instance, two office scenarios with different PLEs, produced the same k due to their configurations of size and layout. Furthermore, the multipath fading effects could help TCPs to set k to smaller values at the expenses of increasing the energy consumption. Most importantly, these findings improve on previous results of the k-neighbors connectivity problem based solely on flat scenarios or performed analytically.

As further works, we plan to extend our analysis to mobile networks in 3D scenarios, using different antenna heights, and to aggregate other radio irregularities besides multipath interference.

Acknowledgments

The authors would like to thank CAPES/PROAP for its support.

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