Efficient Routing and Wavelength Assignment for Wavelength-Routed Optical Networks by a GRASP Heuristic

Roger Souza de Paula[1], **Carlos Magnus Carlson Filho**[2]

[1]Departamento de Telemática
Faculdade de Engenharia Elétrica e de Computação (FEEC)
Universidade Estadual de Campinas (Unicamp) – Campinas, SP – Brasil

[2]Faculdade de Tecnologia (FATEC) – São José do Rio Preto, SP – Brasil

`rogersp@dt.fee.unicamp.br, cmfcarlson@terra.com.br`

Abstract. *Optical networks based on wavelength-division multiplexing (WDM) techniques, called wavelength-routed optical networks (WRONs), seem to be the most interesting to constitute the backbones WANs (wide area networks). We consider in this paper the optimized design of virtual topologies on a WRON. We present an exact mixed integer linear programming (MILP) formulation, which encompasses choice, route and traffic intensity setting for each of the lightpaths in the obtained topology. The objective is to minimize the average packet hop distance. The problem formulation can be used to design a balanced network, in which the utilization of both transceivers and wavelength is maximized.*

We solve the linear formulation for small examples, and demonstrate the tradeoff between number of wavelengths transceivers and average packet hop distance. For large networks, we propose a GRASP-based algorithm. GRASP (Greedy Randomized Adaptive Search Procedure) is a supporting concept for a family of heuristics. We customize the procedure in order to fit the WRON design problem. Heuristic results are compared with those obtained by relaxing the integer constraints in the linear formulation. We show that the proposed heuristic procedure efficiently provides high-quality solutions.

1. Introduction

1.1. Optical Networks

The current technological development paradigm creates a society which requires full access to information at any moment, in any place, under any format. Information is carried through worldwide communications networks. Though the voice traffic continues to grow, the increase of data traffic is remarkable.

However, most of the network implementations are not able to provide high-speed broadband services. Fiber-based technologies may address this problem. Indeed, optical fibers have many advantages [Mukherjee 2003], [Mukherjee 2000]: 1) large bandwidth (almost 50 terabits per second); 2) low signal attenuation (around 0.2 dB/km); 3) low distortion; 4) low power consumption; and 5) low cost.

Networks which utilize WDM techniques are known as *wavelength-routed optical networks* (WRONs) [Ramaswami and Sivarajan 2002]. In these networks, each single channel operates at electronic speed, but a lot of simultaneous channels are used. Thus, bandwidth and delay requirements are guaranteed to be achieved.

On the other hand, new questions arise along with photonics. Facilities location (both equipment and infrastructure), topology choice and efficient wavelength allocation are important problems that have to be modeled and solved in order to allow the optimized evolution of the telecommunications systems towards all-optical networks. Such questions concern both operating companies and equipment vendors. The financial investment is significant. Besides, WRON planning is a complex task, because: 1) there are many network architectural options; 2) technological evolution is fast-paced; 3) it is difficult to foresee the cost behavior of the infrastructure elements; 4) some variables are not easy to quantify (e.g., operation and maintenance cost); and 5) telecommunication experiences a deregulation era.

For these reasons, a WRON planning methodology should take into account the formulation of mathematical models which generate information for techno-economical analyses and support decisions under competitive settings.

1.2. Problem Definition

The WDM technique divides the (huge) bandwidth capacity of a single fiber in several non-interfering wavelengths (i.e., WDM channels). Therefore, each network node needs to operate at the speed of one individual channel, which may be, for instance, the maximum electronic speed (a few gigabits per second). Each optical channel (the lightpath) can be used to carry data packets through many fiber links without the need of electronic conversion at the intermediary nodes.

While an ideal solution, to an N-node network, is to establish lightpaths for all the $N(N-1)$ node pairs, in a practical situation some node pairs have to utilize a sequence of lightpaths, which intermediary nodes are equipped with electrical cross-connects and OEO converters, in order to switch among the lightpaths. Packets are said to run a multi-hop path. Due to the high cost of conversion devices, some studies considered sparse wavelength conversion [Iness and Mukherjee 1999], in which a small portion of nodes has wavelengths converters. Wavelength assignment is also a problem, because distinct traffic must not share any wavelength channel in a fiber.

This paper addresses both the routing and the wavelength assignment problems (RWA) by stating an optimization mathematical model whose objective is to minimize the whole amount of multi-hop paths.

1.3. Previous Work

In this section, we mention some papers of relevance for the optimized design of virtual topologies on wavelength-routed optical networks and the use of GRASP (*Greedy Randomized Adaptive Search Procedure*) in a telecommunication problem.

In [Krishnaswamy and Sivarajan 2001a] the authors formulate the virtual topologies design problem over a WRON without wavelength changers. The objective is to minimize the congestion. The formulation is linearizable (an integer 0-1 program); it is solved exactly for small networks. For larger networks, the relaxation problem provides a lower bound on congestion. Various case studies are presented and the results are compared with another paper [Ramaswami and Sivarajan 1996] of the same authors.

In [Banerjee and Mukherjee 2000] the same problem is formulated as a linear program whose objective is to minimize the average packet hop distance. All nodes have full conversion capability (no wavelength continuity restriction). The model works well for balanced networks and networks with dense physical topology. The condition of wavelength clash (i.e., that two or more lightpaths must have different wavelengths channels at the same link) could not be introduced as since it would turn the problem nonlinear. Others works on the subject are reported in [Krishnaswamy and Sivarajan 2001b] and [Mukherjee et. al. 1996].

An iterative algorithm for the virtual topology design (VTD) is proposed in [Karcius et. al. 2005] to maximize the traffic scaling of the optical network accommodating both static and dynamic traffic demands. The authors based their work in [Ramaswami and Sivarajan 1996] without wavelength conversion.

1.4. Contribution of This Work

This paper presents a new model to the WRON design problem. The mathematical formulation is stated as a *mixed integer linear program* (MILP). Binary decision variables indicate the RWA solution (i.e., both the virtual topology and the lightpaths routing), and real variables correspond to the traffic flow level through the network.

The objective of the optimization model is to minimize the average hop distance. Like one related paper [Banerjee and Mukherjee 2000], this function is posed under a linear form. However, unlike in [Krishnaswamy and Sivarajan 2001a], there are no integer (non-binary) variables. This way, the formulation is more suitable for deriving heuristic procedures based on 0-1 decisions. The conventional set of conditions is enhanced by a new group of wavelength collision linear constraints, which were early written as non-linear expressions [Banerjee and Mukherjee 2000].

Whereas MILP formulations can be directly applied to small size networks, their utilization on medium and large instances are generally impracticable under the current computing resources. This paper deals with the question by proposing a GRASP-like heuristics. The developed algorithm fulfills all the problem requirements, as well as the MILP does: the same input data are expected and the same type of result is produced. It runs independently, i.e., it is neither part of the MILP nor coupled with any other algorithm.

To the best of the authors' knowledge, a GRASP approach to the WRON design is original. Tests involving the proposed heuristics are reported throughout the paper. The procedure has proven to work fast. Moreover, results are consistently close to known bounds of the examples' objective values.

The remainder of the paper is organized as follows. Section 2 defines the parameters and variables of the problem along with the MILP formulation. Section 3 introduces the GRASP concept and presents in detail its customization to support WRON design decisions. Section 4 reports the tests performed in order to validate the MILP on a small network. In the same section, the GRASP algorithm is applied to larger networks. Performance and qualitative comparisons are also provided. Finally, concluding remarks are given in Section 5.

2. Problem Formulation

We formulate a new optimization model based on [Krishnaswamy and Sivarajan 2001a], [Banerjee and Mukherjee 2000], [Krishnaswamy and Sivarajan 2001b] for the design of virtual topology on a WRON. The model is stated as a mixed integer linear program and utilizes multi-commodity flow principles for choosing the constituent lightpaths, establishing the physical support and determining the traffic flow level for each lightpath. In order to make associations with others works easy, we adopt a notation as close as possible to [Krishnaswamy and Sivarajan 2001a] and [Banerjee and Mukherjee 2000]. However, for the sake of completeness, we give detailed description of the problem elements. The following notation is utilized:

s, d	source and destination of a packet, respectively;
i, j	originating and terminating node of a lightpath (virtual link), respectively;
q	qth multiple virtual link between nodes terminating of lightpath;
l, m	endpoints of a physical link;
k	wavelength number.

A. Parameters

N	number of nodes in the network;
$\Lambda^{(s,\,d)}$	traffic matrix. It represents the average rate of traffic flow (in packets/s) from node s to node d, with $\Lambda^{(s,\,s)} = 0$ for s, $d = 1, 2, ..., N$;
$P_{l,\,m}$	existence of a physical link in the physical topology. If $P_{l,\,m} = 1$ then there is a fiber link between nodes l and m, otherwise $P_{l,\,m} = 0$;
Q	maximum number of virtual links for each node pair;
W	number of wavelengths supported by the fiber;
T_i	number of transmitters at node i ($T_i \geq 1$);
R_j	number of receivers at node j ($R_j \geq 1$);
$d_{l,\,m}$	fiber length matrix. Note that $d_{l,m} = d_{m,l}$, and $d_{l,\,m} = \infty$ if $P_{l,\,m} = 0$. It may be given in time units whether the propagation delay is used as measure.
$D_{s,d}$	shortest path (delay) matrix between nodes s and d;
α	lightpath length bound, with $1 \leq \alpha < \infty$;
C	capacity of each lightpath (usually in packet/s);
β	maximum loading lightpath, with $0 \leq \beta \leq 1$.

B. Variables

1) Virtual link variable: $b_q(i, j) = 1$, if there exists a qth multiple virtual link or directed edge (i, j), q in the virtual topology; else $b_q(i, j) = 0$.

2) Wavelength assignment variable: $C_{l,m}^{(k,q)}(i,j)=1$, if the qth multiple virtual link between nodes i and j uses wavelength k and is routed through physical link (l, m); else $C_{l,m}^{(k,q)}(i,j)=0$.

3) Traffic intensity variable: $\lambda_{(i,j),q}^{(s,d)}$ denotes the traffic intensity on the qth multiple virtual link (i, j) for traffic between source-destination pair (s, d).

C. Objective

$$\min \frac{1}{\sum_{s,d}\Lambda^{(s,d)}}\sum_{i,j}\sum_{s,d}\lambda_{(i,j),q}^{(s,d)} \tag{1}$$

Remark: The objective function minimizes the average packet hop distance in the network. Note that $\Sigma_{s,\,d}\,\Lambda^{(s,\,d)}$ is a constant for a traffic matrix in particular.

D. Constraints

1) Virtual link degree:

$$\sum_{q}\sum_{j} b_q(i,j) \le T_i, \qquad \forall\ i \tag{2}$$

$$\sum_{q}\sum_{i} b_q(i,j) \le R_j, \qquad \forall\ j \tag{3}$$

where $b_q(i, j) \in \{0, 1\}$, $(i, j) \in \{1, 2, ..., N\}$ and $q \in \{1, 2, ..., Q\}$.

Remark: The above constraints ensure that the amount of transmitters (receivers) is an upper bound on the number of virtual links originated (terminated) at node i (j).

2) Wavelength:

a) *Wavelength Clash*

$$\sum_{q}\sum_{i,j} C_{l,m}^{(k,q)}(i,j) \le 1, \qquad \forall\ (l, m) \text{ e } k \tag{4}$$

where $C_{l,m}^{(k,q)}(i,j) \in \{0, 1\}$, $(l, m) \in \{1, 2, ..., N\}$ and $k \in \{1, 2, ..., W\}$.

Remark: We ensure here that two or more virtual links traversing through the physical link (l, m) never will be assigned the same wavelength, i.e., there is no wavelength clash.

b) *Conservation of Wavelength*

$$\sum_{k}\sum_{l} C_{l,m}^{(k,q)}(i,j)P_{l,m} - \sum_{k}\sum_{l} C_{m,l}^{(k,q)}(i,j)P_{m,l} = \begin{cases} b_q(i,j), & \text{if } m=j \\ -b_q(i,j), & \text{if } m=i, \quad \forall\ (i,j),\ q \text{ and } m \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Remark: The above equation assures that a wavelength is conserved at every node for a virtual link $b_q(i, j)$ in the physical topology. Observe that this equation is similar to flow conservation equations in multi-commodity flow problems.

3) Number of wavelengths:

$$\sum_{k}\sum_{i,j} C_{l,m}^{(k,q)}(i,j) \le WP_{l,m}, \qquad \forall\ (l, m) \text{ and } q \tag{6}$$

Remark: The above inequality ensures that the number of lightpaths flowing through a fiber link does not exceed *W*.

4) Traffic:

a) *Traffic Routing*

$$\lambda^{(s,d)}_{(i,j),q} \le b_q(i,j)\Lambda^{(s,d)}, \qquad \forall\ (i,j),\ (s,d) \text{ and } q \tag{7}$$

Remark: The previous inequality ensures that the traffic can only flow through an existing lightpath.

b) *Flow Conservation*

$$\sum_q \sum_j \lambda^{(s,d)}_{(i,j),q} - \sum_q \sum_j \lambda^{(s,d)}_{(j,i),q} = \begin{cases} \Lambda^{(s,d)}, & \text{if } i = s \\ -\Lambda^{(s,d)}, & \text{if } i = d, \\ 0, & \text{otherwise.} \end{cases} \quad \forall\ (s,d) \text{ and } i \tag{8}$$

Remark: This is a flow conservation equation at each node for the traffic between nodes *s* and *d* in the virtual topology. Observe that the routing traffic from a given source-destination pair (*s*, *d*) may be bifurcated.

5) Capacity of the lightpath:

$$\sum_{s,d} \lambda^{(s,d)}_{(i,j)} \le b_q(i,j)\beta C, \qquad \forall\ (i,j) \text{ and } q \tag{9}$$

Remark: The inequality above restricts the capacity of the lightpath (channel) in the formulation.

6) Lightpath length:

$$\sum_{l,m} C^{(k,q)}_{l,m}(i,j)\, d_{l,m} \le \alpha D_{i,j}, \qquad \forall\ (i,j),\ k \text{ and } q \tag{10}$$

Remark: This above constraint is optional, and may be incorporated in the formulation to bound delays in the network, so that it avoids long and convoluted lightpaths.

3. The Heuristic Procedure

The *Greedy Randomized Adaptive Search Procedure* (GRASP) [Resende and Ribeiro 2003] is a supporting concept for a family of heuristic optimization procedures. It searches solutions for combinatorial problems by applying a mix of both greedy and random steps. While greedy steps allow feasible solutions to be quickly obtained, randomness avoids the search procedure to be trapped in a local sub-optimum.

GRASP-like algorithms have two major stages: a *construction phase* (which produces a feasible solution) and a *local search phase* (which seeks to improve the quality of the solution by inspecting its neighborhood). The best of all found solutions is kept. Figure 1 presents the generic GRASP pseudo-code.

At the construction phase, a feasible solution is iteratively produced. In order to choose the element that will join the (partial) solution, a Restricted Candidate List (RCL) is built. In the RCL, candidates are sorted according to the contribution they give to the objective function. This contribution is called the *incremental cost*. Elements are randomly picked up from the RCL. Thus, different solutions may be obtained for the same candidate set. When an element is withdrawn from the RCL, incremental costs are re-evaluated, thus giving the adaptive feature to the method.

There are several ways of building the RCL as well as of selecting an element from the RCL. In this work, we adopted a GRASP enhancement for the construction mechanism: the *heuristic-biased stochastic sampling* [Bresina 1996]. However, the produced solution is not guaranteed to be a local optimum. It is then straightforward to perform a local search phase in order to improve the solution quality. Again, distinct local search methods can be used.

In the sequel, it is shown how the GRASP concept has to be customized to be useful to the WRON planning problem.

3.1. Construction Phase

At the construction phase, routes for each origin-destination pair are set one at a time. The RCL contains exactly *n_RCL* elements. The RCL elements are the non-routed origin-destination pairs (s,d), sorted by their demands in descending order. At any iteration, an element is selected with probability $\pi(l) = r_l \Big/ \sum_{k \in RCL} r_k$.

```
procedure GRASP ( )

    Get_Input_Data ( );
    for k = 1, . . . , MAX_ITER do
        solution ← Construction_Phase ( );
        solution ← Local_Search_Phase (solution);
        Update_Best_Solution (solution, best_solution);
    end_for
    return best_solution;

end GRASP.
```

Figure 1. Pseudo-code of the GRASP metaheuristics.

The algorithm has three major modules. First, traffic demands between any (s,d) pair are routed by using the shortest path, in one single virtual hop (as long as possible). Constraints are the amount of available transceivers at both origin and destination nodes, the wavelength availability and the capacity of the physical links which support the lightpath. At the end of the first module, a partial single-hop topology is established. The second module works on the nodes that are not connected yet. By using the shortest path, each disconnected node is linked to another randomly selected node, subject to constraints on the amount of available transceivers and wavelengths. The module adds new unused (candidate) virtual links to the partial solution. The third module finds the minimum-hop path for each non-routed demand, given the capacity constraint on the involved physical links. No new virtual link is created in this module. The pseudo-code for the construction phase is shown in Figure 2.

```
procedure Construction_Phase ( )
    solution ← ∅;
    attempts ← 1;
    while attempts <=MAX_ATTEMPTS do
        Sort $(s, d)$ pairs according to descending traffic demand;
        Build the Restricted Candidate List (RCL);
        while the RCL is not empty do
            Randomly select an element $(s, d)_t$ from the RCL;
            Find the shortest path between nodes $s$ and $d$;
            if there are transceivers, wavelengths and capacity available then
                Route the $(s, d)_t$ element through the shortest path;
                Update the virtual topology;
                solution ← solution ∪ { $(s, d)_t$ };
            end_if
        end_while
        if there is a node $u$ disconnected then
            Randomly select a node $v \neq u$;
            if there are transceivers and wavelengths available then
                Set a lightpath between nodes $u$ and $v$;
                Update the virtual topology;
            else
                Update the set of non–routed pairs;
            end_if
        end_if
        while there is a non–routed $(s, d)$ traffic do
            Randomly select a non–routed $(s, d)$ pair;
            Find the shortest path between nodes $s$ and $d$;
            if there is available capacity then
                Route the $(s, d)$ traffic through the shortest path;
                solution ← solution ∪ { $(s, d)$ };
            end_if
        end_while
        attempts ← attempts +1;
    end_while
    return solution;
end Construction_Phase.
```

Figure 2. Construction phase algorithm.

The construction phase may not succeed in routing all the traffic demands. In this case, the incomplete solution is discarded and the procedure restarts. The construction phase has *MAX_ATTEMPTS* opportunities to obtain a complete feasible solution. If this does not happen, the problem is declared to be infeasible.

3.2. Local Search Phase

In the solution obtained at the construction phase, each origin-destination pair carries its traffic through one or more lightpaths. The local search phase attempts to decrease the objective value by finding, for each (*s*,*d*) pair, a new way of routing the demand such that the amount of lightpaths is reduced. First of all, the procedure verifies if the total amount of virtual links is lesser than the maximum ($N*\Delta$, where N is the number of network nodes and Δ is the number of transceivers), which is the condition for being able to try new routing options. A new routing option is interesting if it improves the whole solution (i.e., the objective value is decreased). The local search is performed until no improvement is possible. The pseudo-code for the local search phase is shown in Figure 3.

```
procedure Local_Search_Phase (solution)

    if the current number of virtual links <= $N*\Delta$ then
        Sequentially select a $(s, d)$ pair;
        Find the shortest path between nodes $s$ and $d$;
        if there are transceivers and wavelengths available then
            Create a virtual virtual between $s$ and $d$;
            Update the virtual topology;
        end_if
    end_if
    for each $(s, d)$ pair do
        Find the shortest path between nodes $s$ and $d$;
        if there is available capacity then
            Set a candidate route through the shortest path;
            Calculate the incremental cost of using the candidate route;
        end_if
        if the incremental cost of the candidate route < incremental cost of the current route then
            Route the $(s, d)$ traffic through the candidate route;
            solution ← solution – current $(s, d)$ route + candidate $(s, d)$ route;
        end_if
    end_for
    return solution;

end Local_Search_Phase.
```

Figure 3. Local search phase algorithm.

4. Numerical Results

This section presents numerical results for some studied scenarios for the design of virtual topologies problem on a WRON. We consider in this paper a small network and a moderately large network. We will see that the number of variables and equations grows quickly as the number of nodes and edges increases.

In each of the scenarios, the links in the physical topologies are bidirectional, i.e., there is a pair of unidirectional fibers, one in each direction. Each routing node is equipped with a WRS (wavelength-routing switch) and many transceivers. We assume that we have full wavelength conversion capability at the nodes where there are wavelength converters. This means that we do not need wavelength continuity constraints. The number of transmitters is assumed to be equal to the number of receivers. For all tests realized here, we utilize the number of the transceivers and wavelengths with values between 1 and 10.

The formulated MILP is NP-hard. The number of variables and constraints grow approximately as $O(N^4)$. Hence, to obtain the exact solution may be computationally intractable. We denominate ***LP-relaxation*** of the MILP the resulting LP where the variables' integrality constraints are relaxed.

We utilize the mathematical programming language MPL [Maximal] and the branch-and-bound method available in the CPLEX® optimization software [ILOG] for, respectively, constructing the model and solving the MILP and the LP-relaxation.

The GRASP algorithm was codified in the C programming language. As the GRASP is a heuristics which has a probabilistic component, it would be reasonable to execute it a pre-determined number of times *TIMES*. Thus, we run the algorithm a certain number of times, and we present the best and the average of all obtained solutions.

We also study the characteristics of resource utilization for the largest networks presented here. The utilization of the transceivers and wavelengths plays an important role for the network design: a network with many transceivers and few wavelengths may lead to a high number of unused transceivers (wavelength constraints), and vice-versa. Balancing the available network resources is a way to obtain a better usage of transceivers and wavelengths. In the following, the obtained results for each of the studied scenarios are exhibited.

4.1. Six-Node Network

The MILP was solved exactly for an arbitrary six-node network (Figure 4). The traffic matrix considered for this network is shown in Table 1. Each traffic entry was generated randomly from a uniform distribution in [0, 1]. We also used as input parameters the number of transceivers Δ (the network *degree*), the number of wavelengths W and the number of maximum virtual links Q allowed for each node pair. In general, for all the scenarios, we expected that the larger the values of the parameters, the smaller the average packet hop distance, since the problem will be "looser", that is, there is a greater availability of resources.

The results are reported in Table 2. The parameters are represented by the first three columns, respectively. The column designated by LB denotes the LP-relaxation, i.e., the lower bound on the MILP objective value (minimization problem). The MILP column refers to the exact solution obtained for the problem. The asterisk (*) implies that the corresponding column parameter is not constrained in the respective scenario. We present some results for several possible combinations of these three parameters. The remaining utilized parameters are $\alpha = 1$, $C = 15$, and $\beta = 0.6$. Observe that for degrees higher than five ($\Delta > 5$), the results keep unaltered.

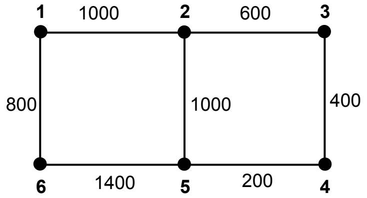

Figure 4. Example of a six-node WAN.

Table 1. Traffic matrix for the six-node network

0.000	0.562	0.813	0.855	0.834	0.295
0.679	0.000	0.448	0.532	0.610	0.583
0.194	0.682	0.000	0.552	0.355	0.535
0.391	0.282	0.988	0.000	0.311	0.659
0.170	0.231	0.529	0.711	0.000	0.636
0.273	0.163	0.017	0.910	0.516	0.000

Table 2. Results for the six-node network

Δ	W	Q	LB	MILP
1	1	*	1.800	2.663
1	*	*	1.800	2.663
2	1	*	1.542	1.542
2	2	*	1.481	1.481
2	*	*	1.481	1.481
3	1	*	1.542	1.542
3	2	*	1.304	1.319
3	3	*	1.276	1.276
3	*	*	1.276	1.276
4	1	*	1.542	1.542
4	2	1	1.262	1.275
4	2	2	1.257	1.275
4	3	*	1.110	1.110
4	4	*	1.103	1.103
4	*	*	1.103	1.103
5	1	*	1.542	1.542
5	2	1	1.259	1.263
5	2	2	1.257	1.263
5	3	*	1.080	1.084
5	4	*	1.000	1.000
5	*	*	1.000	1.000

In Figure 5(a) it is shown the virtual topology obtained by the exact solution of the MILP for the settings $\Delta = 1$, $W = 1$, and $Q = 1$ (actually, Q is unconstrained, i.e., the same result is valid for $Q > 1$). In such situation, the average packet hop distance h_m is 2.663. In the topology, if the node 2 wants to send data to node 3, then the packet will have optical-electrical-optical conversion at nodes 6, 1, 5 and 4 consecutively. Note that in this particular case, only two WRSs are necessary, one at node 1 and other at node 2 (this result points out the possible use of sparse wavelength conversion).

(a) $\Delta = 1$, $W = 1$ and $Q \geq 1$, with $h_m = 2.663$.

(b) $\Delta = 2$, $W = 1$ and $Q \geq 1$, with $h_m = 1.542$.

(c) $\Delta = 2$, $W = 2$ and $Q = 1$, with $h_m = 1.481$.

(d) $\Delta = 2$, $W = 2$ and $Q = 2$, with $h_m = 1.481$.

Figure 5. Some virtual topologies from obtained results.

In Figure 5(b) it is exhibited the resulting virtual topology form the resolution of MILP for the settings $\Delta = 2$, $W = 1$, and $Q \geq 1$. The average packet hop distance is 1.542. Observe, once again, that the Q does not interfere in the solution.

Next, by increasing the number of wavelengths W to two, and keeping all others parameters as in Figure 5(b), we obtain two distinct topologies. They are shown in Figure 5(c) and Figure 5(d) for $Q = 1$ and $Q = 2$, respectively. Despite the average packet hop distance is the same, the virtual topologies are slightly different (see the physical links used by the lightpath between nodes 3 and 6).

A final and important observation: the GRASP algorithm was also applied for this network, and systematically achieved the optimal solution. Thus, we left the discussion of the heuristics to the next topic.

4.2. NSFNET

This scenario the National Science Foundation Network [Banerjee and Mukherjee 2000], which has 14 nodes and 21 edges (see Figure 6). The traffic matrix is randomly generated by taking, for 56 node pairs (an average of four per node, or about 30% of the matrix entries), values from an uniform distribution in $[0, C\delta/a]$. The remaining traffic uses a uniform distribution between $[0, C/a]$, where C is the capacity of the lightpath, a is an arbitrary integer and δ is the traffic intensity. The matrix is shown in Table 3. Note that this way of generating the traffic matrix is the same as presented in [Banerjee and Mukherjee 2000], although the matrix entries probably have different values. Therefore, the obtained results may be qualitatively compared.

For the tests we use $C = 1250$, $a = 20$, $\delta = 10$, $\beta = 0.8$, $\alpha = 2$ and $Q = 1$ (i.e., multiplicity is not allowed). The results are exhibited in Table 4. At the first part of the table, results of the LP-relaxation are listed. The symbol "*" indicates that for a higher number of wavelengths, the solution persists equal. The first entry of the table (in the northwest corner) indicates the first time the problem becomes feasible. Thus, the LP-relaxation is infeasible for $\Delta < 3$ and/or $W < 2$. For the NSFNET, the GRASP algorithm is run with *TIMES* = 100 and *MAX_ITER* = 100. The best solution is shown in the second part of Table 4, while the average solution obtained is presented in the third part.

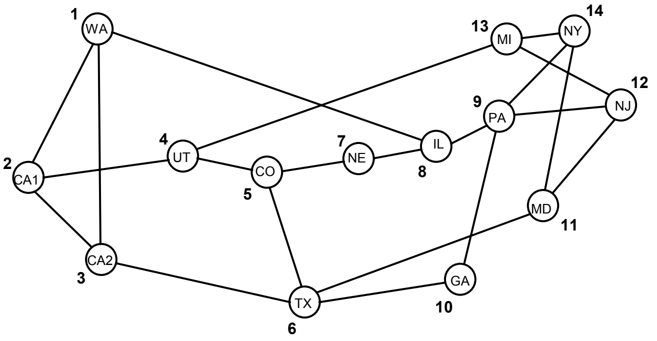

Figure 6. National Science Foundation Network.

Table 3. Traffic matrix for the NSFNET

0.000	31.633	24.901	58.240	35.050	211.240	321.889	0.468	55.841	50.833	276.879	33.442	5.561	24.889
40.610	0.000	66.543	48.191	153.232	32.075	39.772	32.207	1.991	35.593	24.711	298.190	50.550	17.901
53.526	57.702	0.000	301.288	428.238	437.050	14.330	32.152	192.457	7.905	51.262	4.932	23.695	30.074
241.097	42.593	7.675	0.000	206.411	24.535	35.744	606.209	28.720	57.529	39.436	52.695	54.295	58.967
37.346	8.356	53.575	31.746	0.000	18.653	54.101	327.549	351.772	53.667	17.281	17.384	457.725	39.504
45.704	32.200	449.007	50.553	1.723	0.000	22.953	55.018	54.468	27.552	188.099	54.913	59.856	180.513
61.823	324.573	268.376	5.634	61.038	4.721	0.000	58.670	609.545	203.159	37.573	23.118	57.170	52.439
31.760	11.637	441.210	32.781	26.332	45.357	5.167	0.000	11.911	29.068	44.177	602.327	55.072	41.605
35.683	52.618	27.589	19.318	4.673	241.187	52.685	3.407	0.000	562.542	61.260	559.846	59.226	83.030
65.193	40.782	31.018	249.953	26.764	15.889	40.487	544.316	47.839	0.000	34.993	211.251	54.515	29.139
239.586	34.609	9.801	11.843	485.934	426.834	51.930	12.720	19.003	21.419	0.000	118.273	33.200	32.141
327.852	2.893	1.359	17.249	31.683	49.381	25.966	13.454	223.786	35.653	603.005	0.000	562.916	19.788
9.163	121.345	43.697	134.634	18.318	40.573	32.157	22.054	529.453	111.723	10.489	5.431	0.000	51.394
423.971	20.028	10.989	161.213	17.283	7.014	47.448	56.683	47.508	281.653	0.173	28.205	18.620	0.000

The GRASP results may be qualitatively compared with the heuristics *Maximizing Single-Hop Traffic* and *Maximizing Multi-hop Traffic* proposed in [Banerjee and Mukherjee 2000]. It should be remembered that, in [Banerjee and Mukherjee 2000], the mathematical formulation does not include the inequalities group (4) (see Section 2), which would turn the problem nonlinear and, usually, more difficult to solve. By omitting these constraints (which may actually restrict the solution space), the authors have made an approximation to a WRON without clash wavelength. Further, the authors, differently from the approach we adopted in this paper, obtain an exact (non-optimal) solution by executing only two iterations of the branch-and-bound algorithm, what leads to more realistic solutions. The Maximizing Single-Hop Traffic and the Maximizing Multi-hop Traffic heuristics seek to improve the (already) exact solution. Taking these factors into consideration, one can expect that the results presented in [Banerjee and Mukherjee 2000] went better than those shown here. However, in many times, GRASP results are dominant. Since GRASP is an independent procedure, we conclude it is quite adequate to the WRON design problem.

Table 4. Results for the NSFNET

LP-Relaxation Solution

Transceivers (Δ)	Wavelengths (*W*)								
	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3	1.426	1.365	*						
4	1.384	1.244	1.226	*					
5	1.380	1.208	1.164	1.161	*				
6	1.380	1.202	1.143	1.127	*				
7	1.380	1.202	1.138	1.107	1.097	*			
8	1.380	1.202	1.137	1.100	1.078	1.070	*		
9	1.380	1.202	1.137	1.099	1.072	1.054	1.049	*	
10	1.380	1.202	1.137	1.099	1.070	1.048	1.034	1.030	*

GRASP Final Solution

Transceivers (Δ)	Wavelengths (*W*)								
	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3	1.646	1.497	1.406	1.392	1.384	1.390	1.384	1.390	1.390
4	1.567	1.394	1.302	1.254	1.237	1.236	1.236	1.234	1.236
5	1.547	1.349	1.257	1.204	1.179	1.168	1.163	1.162	1.162
6	1.520	1.339	1.239	1.180	1.152	1.141	1.134	1.129	1.128
7	1.513	1.335	1.230	1.164	1.136	1.122	1.115	1.107	1.102
8	1.513	1.335	1.229	1.160	1.128	1.108	1.097	1.090	1.084
9	1.513	1.335	1.230	1.161	1.124	1.102	1.089	1.079	1.071
10	1.513	1.335	1.231	1.160	1.121	1.100	1.082	1.071	1.062

GRASP Average Final Solution

Transceivers (Δ)	Wavelengths (*W*)								
	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3	1.681	1.524	1.421	1.397	1.394	1.394	1.394	1.394	1.394
4	1.587	1.407	1.307	1.258	1.241	1.239	1.238	1.238	1.238
5	1.561	1.362	1.260	1.205	1.183	1.172	1.164	1.163	1.163
6	1.544	1.350	1.244	1.183	1.155	1.145	1.135	1.131	1.129
7	1.544	1.346	1.232	1.166	1.138	1.125	1.116	1.109	1.103
8	1.543	1.346	1.232	1.161	1.131	1.112	1.100	1.092	1.086
9	1.544	1.346	1.232	1.161	1.125	1.104	1.089	1.080	1.073
10	1.543	1.346	1.232	1.161	1.124	1.101	1.084	1.072	1.064

As previously mentioned, we also analyze the impact of the utilization of transceivers and wavelengths on network cost. Transceivers and wavelengths are determinant factors for the cost of terminating and switching equipment, respectively. The transceivers and wavelengths average utilization are drafted in Figure 7. By comparing the results in Table 4 with the figures, we see that six transceivers and four wavelengths is a good choice for the NSFNET design with this particular traffic pattern, since we achieve a very reasonable average packet hop distance and almost 100% of resources utilization (transceivers and wavelengths).

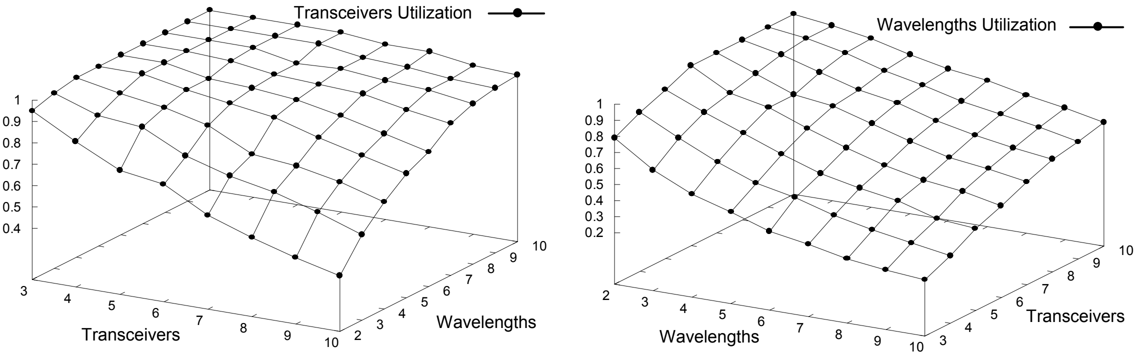

Figure 7. Average transceivers and wavelengths utilization for the NSFNET obtained by GRASP.

We graphically compare, for a six-transceiver network, the results of GRASP algorithm achieved with the LP-relaxation bounds (Figure 8). As expected, the average packet hop distance decreases while the number of wavelengths available on fiber is increased.

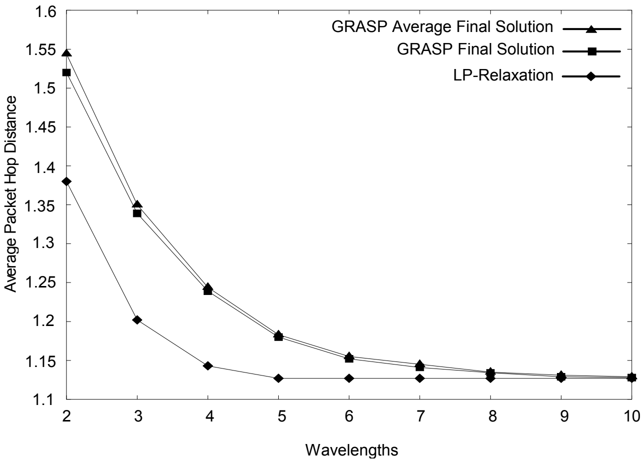

Figure 8. Comparison of GRASP with LP-relaxation for the NSFNET with six transceivers.

5. Conclusions

This paper presented a mixed integer linear programming model (MILP) for the virtual topology design problem on a wavelength-routed optical network. The objective is to minimize the average hop length of a lightpath in the absence of wavelength continuity constraints. The mathematical formulation, when solved, provides a complete solution for the logical virtual design problem.

For small networks, we could solve the MILP exactly. In the six-node network example, we discussed the relationship among the number of transceivers in each node, the number of wavelengths of fiber and the average packet hop distance. However, for larger networks like the NSFNET, we needed to use a heuristic algorithm for the topology design. Thus, we proposed a GRASP approach for the problem and demonstrated its good performance while compared to the relaxed solution (a lower bound to the problem). As far as we know, this approach is unedited for this kind of problem. We saw that the heuristic solutions were very close to the ones obtained by LP-relaxation, especially for the cases where the designed virtual topology had a higher number of transceivers (high degree).

We believe that wavelength-routed optical networks tend to be dominant in the near future. They will grow in size and complexity. In consequence, exact planning models will have their applicability limited. Thus, heuristic procedures like the one presented in this paper will become more and more important. GRASP has proved to be a comprehensive and efficient approach for the virtual topology design.

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