ILP Formulation and *K-Shortest Path* Heuristic for the RWA Problem with Allocation of Wavelength Converters

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***Abstract** – Different integer linear programming (ILP) formulations have been proposed for the routing and wavelength assignment (RWA) problem in WDM networks. An important goal in the design of WDM networks is to use the minimum number of converters to serve communication needs, because this is cost effective. In this paper, we study the problem of allocating the minimum number of converters in a network for solving the RWA problem. Furthermore, our formulation allows any kind (partial or full, sparse or ubiquitous) of wavelength conversion in the network. We also propose an experimental comparison of the heuristic from [Swaminathan and Sivarajan 2002] with a K-shortest path heuristic with limited number of converters, for a large network.*

1. Introduction

The design of an optical network involves three closely related tasks: dimensioning, routing, and wavelength assignment. Given is a physical topology of the network as well as, for every pair of nodes, a demand for a number of lightpaths to be established between these nodes [Koster and Zymolka 2005], [Subramaniam, S. and Azizoglu, M., 1996]

Optical networks design draws an increasing amount of attention nowadays. The Wavelength Division Multiplexing (WDM) technique splits the large bandwidth available in optical fibers into multiple channels, each one operating at different wavelengths and at specific data rates (up to 40Gbps). The lightpaths are logical channels which provide an end-to-end connectivity in the all optical network [Ramaswami and Sivarajan 1998]. The performance of the WDM networks can be improved by allowing wavelength translation (conversion) at the routing nodes [Swaminathan and Sivarajan 2002], [Ramaswami and Sasaki 1998], [Ramamurthy and Mukherjee 1998]. The key issue in the all-optical network design process once the physical realization has been achieved, is to properly dimension the network with respect to wavelength numbers and to determine a wavelength allocation plan [Swaminathan and Sivarajan 2002], [Murty and Gurusamy 2002].

For circuit switched lightpath service on an optical network, each lightpath will carry traffic related to only one source-destination pair. A session is a set of connections between a source-destination pair. For example (A,B:1), (B,C:2) are sessions wherein the number of connections required from node A to node B is one and from node B to node C is two.

A wavelength-convertible network, which supports complete conversion at all nodes, is functionally equivalent to a circuit-switched network, i.e., lightpath requests are blocked only when there is no available capacity on the path. Moreover, in most cases, it may be uneconomic to deploy wavelength conversion capability at all nodes, but having a few nodes with wavelength conversion capabilities may be desirable [Murthy and Gurusamy 2002]. Then the questions are (1) How many nodes in a network should have conversion capability? (2) How many converters should a node have?

In this paper we extend a heuristic from [Assis and Waldman 2004] and focus on question (2). In order to provide an answer, the key point is the formulation of the well-known Routing and Wavelength Assignment-RWA problem in networks with small number of converters per node, as a matter of fact: while previous literature considers conversion with limitless/large number of converters in nodes [Swaminathan and Sivarajan 2002], [Ramaswami and Sivarajan 1995]; our strategy considers modifications in RWA problem to allow allocation of a limited number of converters in a node of the network. It is important because this number, if small, is cost effective. Furthermore, our linear formulation and *K-shortest path* heuristic allow any kind (partial or full, sparse or ubiquitous) of wavelength conversion in network.

The paper is organized as follows. Section 2 defines the traditional RWA problem, extending an auxiliary graph approach for a limited number of converters, and we discuss our results for a ring topology. Section 3, we discuss our results for a mesh topology and explain over the computational complexity. In section 4, we propose a heuristic with limited number of converters for large networks. The heuristic is executed on a random mesh National Science Foundation Network (NSFNET) and the performance is compared with *K-shortest path* heuristic without limited number of converters from [Swaminathan and Sivarajan 2002]. Finally, section 5 presents our conclusions.

2. Routing and Wavelength Assignment Problem

The physical topology network is represented as $G(N,E,W)$, in which N represents the sets of nodes, E represents the set of directional fibers, and W represents the set of wavelengths on each link. The physical topology and the traffic matrix are given as input for the problem. Our objective is maximize the number of lightpaths to be established from the traffic matrix. Since we try to maximize connections in a given session or traffic matrix, for a fixed set of wavelengths, it is called Max-RWA problem [Swaminathan and Sivarajan 2002], [Ramaswami and Sivarajan 1995]. The algorithm that solves the above problem in general should: a) Maximize the number of lightpaths established using the minimum number of wavelengths. In addition, our formulation tries to maximize the number of lightpaths established using the minimum number of converters (If conversion is available). The following assumptions are made in our RWA problem: a) the number of wavelengths in each link of the fiber is assumed to be same b) each call requires a full wavelength on each link of its path c) simplex connections are considered.

2.1 Linear Program

We formulate the Max-RWA problem as a Mixed Integer Linear Program (MILP). In the formulation the paths for a connection are not specified before hand, the linear program solver is allowed to choose any possible path and any possible wavelength for a source–destination pair. Thus the logical topology design and wavelength assignment are inbuilt in the formulation itself, which is not in the formulations presented in [Ramaswami and Sivarajan 1996], but is similar to the presented in [Swaminathan and Sivarajan 2002]. The constraints involve the edges or arcs of the network. By this approach the solution for the RWA problem tends to optimality.

2.2 Definitions

We use the following notation: i and j denote originating and terminating node of a lightpath, respectively; m and n denote endpoints of a physical link.

The parameters of the formulation are: N = Number of nodes in the network. K= The traffic matrix , i.e., K_{ij} is the number of connections that are to be established between node i and j. P_{mn} denotes the existence of a link in the physical topology. Here, P_{mn} = 1, then there is only a fiber link between nodes m and n, otherwise P_{mn} = 0. W denotes the number of wavelengths the fiber can support. $C_l(\varsigma)$: set of the wavelengths into which ς can be converted in node l. $D_l(\varsigma)$: set of the wavelengths that can be converted to ς in node l. The variables of the formulation are: b_{ij} denotes the number of lightpaths established between node i and node j, taking positive integral values. The p^{ij}_{mn} variable denotes the number of lightpaths between nodes i and j being routed through fiber link m-n. L is the maximum number of wavelength channels supported per fiber; $c_{ij\varsigma}$ is the number of lightpaths between node i and node j that start in the wavelength ς; for $\varsigma =1,2,3,...,W$; $d_{ij\varsigma}$ is the number of lightpaths between node i and node j that finish in the wavelength ς; for $\varsigma =1,2,3,...,W$. Furthermore, $p^{ij}_{mn\varsigma}$ = 1, if the lightpath between node i and j uses wavelength ς through physical link m-n.

2.3 Max-RWA: Original Mathematical Formulation

Objective:

$$\textbf{Maximize}: \sum_i \sum_j b_{ij} \tag{1}$$

Remark: The objective here is to maximize the number of connections to be established from the given traffic matrix

Routing on physical topology p^{ij}_{mn} :

$$\sum_m p^{ij}_{mk} = \sum_n p^{ij}_{kn}, \quad if \quad k \neq i, j \tag{2}$$

$$\sum_n p^{ij}_{in} = b_{ij} \quad \forall i, j \tag{3}$$

$$\sum_m p^{ij}_{mj} = b_{ij} \quad \forall i,j \tag{4}$$

$$\sum_{ij} p^{ij}_{mn} \le L.P_{mn} \quad \forall m,n \tag{5}$$

$$b_{ij} \le K_{ij} \quad \forall i,j \tag{6}$$

Remark: These constraints ensure that demand at origin and destination node for a source-destination pair (i,j) is satisfied.

On coloring lightpaths with conversion resources:

$$\sum_n p^{ij}_{in\varsigma} = c_{ij\varsigma} \quad \forall i,j,\varsigma \tag{7}$$

$$\sum_m p^{ij}_{mj\varsigma} = d_{ij\varsigma} \quad \forall i,j,\varsigma \tag{8}$$

$$\sum_\varsigma c_{ij\varsigma} = \sum_\varsigma d_{ij\varsigma} = b_{ij} \quad \forall i,j \tag{9}$$

Remark: These constraints allow a lightpath to be started in wavelength ς, and finished in another wavelength.

$$\sum_m p^{ij}_{ml\varsigma} \le \sum_n \sum_{t\in C_l(\varsigma)} p^{ij}_{lnt} \qquad \text{if } l \ne i,j \tag{10}$$

$$\sum_n p^{ij}_{ln\varsigma} \le \sum_m \sum_{t\in D_l(\varsigma)} p^{ij}_{mlt} \qquad \text{if } l \ne i,j \tag{11}$$

Remark: conversion in intermediate nodes, explained later.

$$\sum_\varsigma p^{ij}_{mn\varsigma} = p^{ij}_{mn} \quad \forall i,j,m,n \tag{12}$$

$$\sum_{ii} p^{ij}_{mn\varsigma} \le P_{mn} \quad \forall m,n,\varsigma \tag{13}$$

Remark: Constraints (12) guarantees that the number of wavelengths present in each physical link is equal to the number of lightpaths traversing it. In (13) we assure that there is no wavelength clash at physical link.

Int $b_{ij}, p^{ij}_{mn}, c_{ij\varsigma}, d_{ij\varsigma}$ **Bin** $p^{ij}_{mn\varsigma}$

2.3.1 Explanantion of (10) and (11)

In a node with resources of conversion, equations (10) and (11) guarantee that a wavelength that arrives in node *l* in color ς can be converted to another wavelength in accordance with definition of the $C_l(\varsigma)$ and $D_l(\varsigma)$ sets. Notice that if conversion is not allowed (fig. 1b) in node *l=4*, then:

$$C_l(\varsigma) = D_l(\varsigma) = \{\varsigma\}, \ \forall\varsigma$$

Therefore, for $\varsigma = \varsigma_1, \varsigma_2, \varsigma_3$:

$$\sum p^{ij}_{ml\varsigma} \le \sum \sum p^{ij}_{lnt} = \sum p^{ij}_{ml\varsigma} \le \sum p^{ij}_{ln\varsigma}$$

and

$$\sum p^{ij}_{ln\varsigma} \le \sum \sum p^{ij}_{mlt} = \sum p^{ij}_{ln\varsigma} \le \sum p^{ij}_{ml\varsigma}$$

$$\Rightarrow \qquad \sum_n p^{ij}_{\ln\varsigma} = \sum_m p^{ij}_{ml\varsigma}.$$

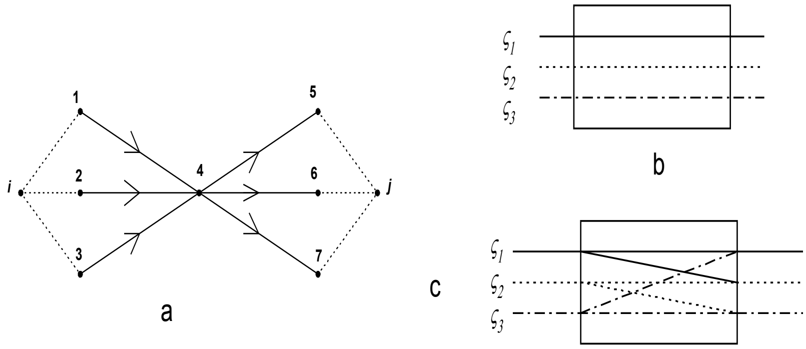


Figure. 1. (a) Optional physical links that arrive and leave node 4 for a lightpath from node i to node j, (b) no conversion, (c) partial conversion.

This last equality expresses a flow conservation of ς-colored paths through node l that is valid when conversion is not allowed. Now, with partial conversion at node l=4, as illustrated for the fig 1c. Then, $C_4(\varsigma_1)=\{\varsigma_1,\varsigma_2\}$, $D_4(\varsigma_1)=\{\varsigma_1,\varsigma_3\}$ and $C_4(\varsigma_2)=\{\varsigma_2,\varsigma_3\}$, $D_4(\varsigma_2)=\{\varsigma_1,\varsigma_2\}$ and $C_4(\varsigma_3)=\{\varsigma_1,\varsigma_3\}$, $D_4(\varsigma_3)=\{\varsigma_2,\varsigma_3\}$. Applying 10 and 11, we get:

$$\Rightarrow p^{ij}_{14\varsigma1} + p^{ij}_{24\varsigma1} + p^{ij}_{34\varsigma1} = p^{ij}_{45\varsigma1} + p^{ij}_{46\varsigma1} + p^{ij}_{47\varsigma1} + p^{ij}_{45\varsigma2} + p^{ij}_{46\varsigma2} + p^{ij}_{47\varsigma2}$$

$$\Rightarrow p^{ij}_{14\varsigma2} + p^{ij}_{24\varsigma2} + p^{ij}_{34\varsigma2} = p^{ij}_{45\varsigma2} + p^{ij}_{46\varsigma2} + p^{ij}_{47\varsigma2} + p^{ij}_{45\varsigma3} + p^{ij}_{46\varsigma3} + p^{ij}_{47\varsigma3}$$

$$\Rightarrow p^{ij}_{14\varsigma3} + p^{ij}_{24\varsigma3} + p^{ij}_{34\varsigma3} = p^{ij}_{45\varsigma1} + p^{ij}_{46\varsigma1} + p^{ij}_{47\varsigma1} + p^{ij}_{45\varsigma3} + p^{ij}_{46\varsigma3} + p^{ij}_{47\varsigma3}$$

Through the example above we evidence that the definition of the sets $C_l(\varsigma)$ and $D_l(\varsigma)$ allows any kind of conversion in network: partial, full, sparse, ubiquitous etc.

2.4 Number of converters

In fig. 2, in order to allow the specification of a limited number of converters, a node with resources of conversion is split into two auxiliary nodes *a* and *b*. After, it is created one unidirectional arc *a-b*. If we would like to have one converter, it is created one more auxiliary node c_1 and two more auxiliary arcs a-c_1 and c_1-b. If we would like to have two converters, it is created one more auxiliary node c_2 and two more auxiliary arcs a-c_2

and $c_2 – b$ and so on. Note that in auxiliary graph constraints arcs “from” or “to” c_i have load L=1 and arcs from a to b don’t have traditional clash constraints. Therefore (5) and (13) must be replaced by two other equations to guarantee the success of the strategy. These two new equations will be shown in the next subsection.

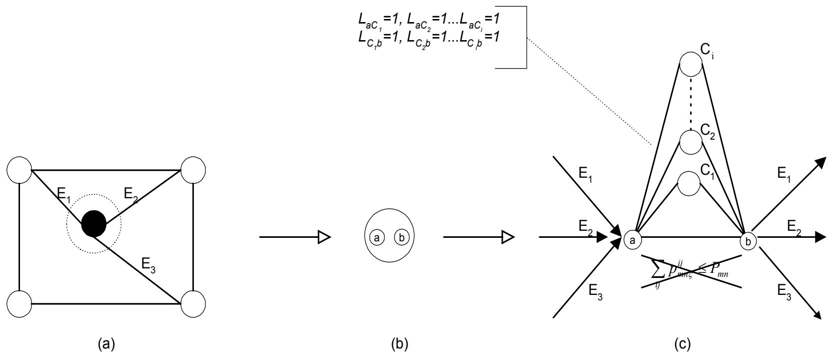


Figure. 2. (a) The original graph with black nodes (with resources of conversion) and bi-directional arcs. (b) Black node is splitting in two nodes (a and b), (c) A auxiliary graph with auxiliary nodes a, b and c: a is incoming of arcs, b is out of arcs and c is an auxiliary node that represents a converter.

2.4.1 Auxiliar Graph (New formulation)

In fig. 2, a, b and c_i denote nodes from the auxiliary graph (for established number of converters N_c with $i=1,2....N_c$). Therefore, if the auxiliary graph is made, i.e., there is the specification of a limited number of converters. Thus, we replace constraints (5) and (13) by the following more restrictive equations and in addition, we define the Nc.

$$\sum_{ij} p_{mn}^{ij} \leq \begin{cases} L.P_{mn} & \text{for (link } m\text{-}n) \neq \text{ (link } a\text{-}c_i) \text{ or} \neq \text{(link } c_i\text{-}b) \\ 1 & \text{for (link } m\text{-}n) = \text{(link } a\text{-}c_i) \text{ or} = \text{ (link } c_i\text{-}b) \end{cases} \qquad (14)$$

$$\sum_{ij} p_{mn\varsigma}^{ij} \leq P_{mn} \quad \text{for (link } m\text{-}n) \neq \text{(link } a\text{-}b) \qquad (15)$$

2.4.2 Complexity

In the formulation with auxiliary graph, based in the strategy of fig. 2, with sparse conversion in one node, the number of nodes grows from N to $N+2$, for 1 converter. From N to $N+3$ for 2 converters and so on. Therefore, the number of nodes grows from N to $N+N_c+1$ nodes, when N_c converters are put in one node of network. Similarly, the number of links E grows from E to $E+3$, for 1 converter, from E to $E+5$ for 2 converters and so on. Therefore the number of links grows from E to $E+1+2.N_c$ when N_c converters are put in one node of the network. If the same number of converters are put in all N nodes of network, then the number of nodes in formulation grows from N to

$2N+(N.N_c)$ and the number of links grow from E to $E+(1+2.N_c).N$. Issues, like the number of routing variables $O(N^2.E.W)$, where there are W wavelengths, will become critical in analyzing scalability. However, in the network practical design the cost is minimized when the minimum number of converters is used [Yiming and Oliver 2003], [Assis and Waldman 2004].

2.4.3 The simple example (Nc is the number of converters in one node with conversion resources)

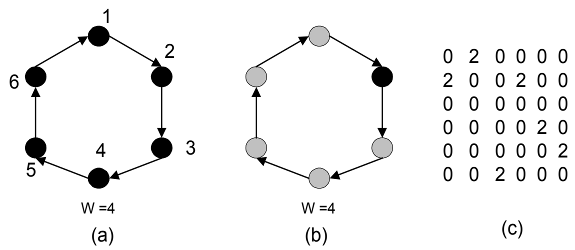


Figure. 3. (a) Nr=6 and (b) Nr=1: two unidirectional rings (clockwise direction). Black nodes-conversion. Clear nodes-no conversion, (c) Traffic matrix (12 connections).

Notice that with the application in formulation/strategy with N_c=2 in one node, fig. 3b, we will obtain 12 connections, what proves the efficiency and cost saving of the formulation. Notice in table 1, that in auxiliary graph, the computational cost increases, see previous subsection, and is given by: Node cost = *N+Nr.(1+Nc)*, Link cost: = *E+Nr.(1+2.Nc)*, where *Nr* is the number of nodes with conversion resources.

Table 1: Computational cost for auxiliary graph

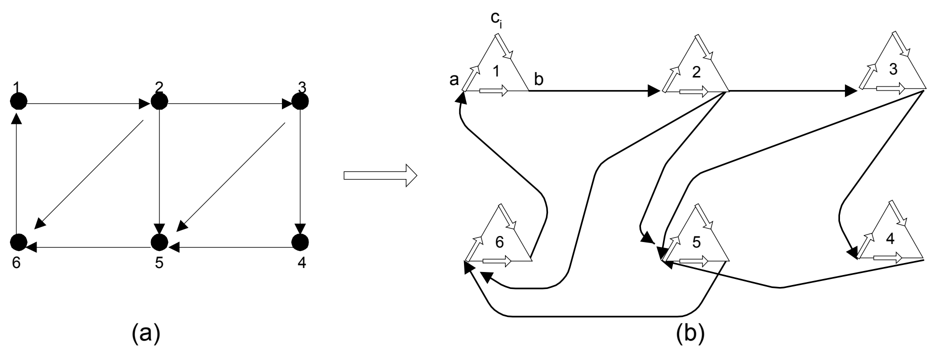
Ring	Nc	Node cost	Link cost computational
(b) *Nr*=1	1	8	9
	2	9	11
	3	10	13
	4	11	15
(a)	1	18	24
	2	24	36
Nr=6	3	30	48
	4	36	60

3. Simulations

An exhaustive approach that enumerates all the possible ways of converter number placement and choosing the best one is not efficient for moderate large networks. In this section, we propose simulations in the following way:

1. To allow full conversion in all nodes of the network in the original graph, to apply the traditional formulation and to get the maximum number of established connections. This way, the RWA suppose that all node has limitless or great number of converters in each node of the network, in agreement with the physical-out-degree of this node. For example, in a node with physical-out-degree δ_p and W wavelengths, the number of converters Nc will allow $Nc = W.\delta_p$ (as it is applied for all nodes, we have ubiquitous conversion). This step serves only for comparison.
2. Using the auxiliary graph (proposed strategy), to place Nc=1 in all nodes of the network. To apply the modified formulation and to verify the established number of connections.
3. If the number of established connections in previous step is not same the number obtained in step one, return to the auxiliary graph (step 2) with $Nc=2$, and so on.

Notice that when we use the auxiliary graph, the connections always have their origin in auxiliary nodes "b" and always finish in auxiliary nodes "a". For example (fig. 4b), connections that are initiated in node "1" in the original graph, are initiated in the node "b" in auxiliary graph and connections which are finished in node "1" in original graph, are finished in the node "a" in auxiliary graph.



Figures. 4.: (a) 6 node-mesh topology (b) auxiliary graph for 6 node-mesh topology.

Table 2. Traffic matrix for 6 node mesh network

0	0	0	0	0	0
0	0	3	3	0	7
0	0	0	0	7	0
0	0	0	0	3	0
3	0	0	0	0	0
0	0	0	0	3	0

3.1 Numerical Results for 6 node-mesh topology

Simulations have been carried out to investigate the performance of the proposed strategy with auxiliary graph over an 6-node mesh topology (Fig.4). The traffic matrix used by this network is shown in table 2. The total number of connections to be set up is 29. The results with Linear Program are tabulated in table 3 for the cases of No conversion/original graph. Full conversion with *Nc*=1/auxiliary graph and Sparse conversion/auxiliary graph with one or two nodes with *Nc*=1.

In table 3, the number of connections obtained with *Nc* = 1 in all nodes is same to the number obtained with ubiquitous conversion with *Nc* limitless, for all wavelength plans; therefore, the latter is not shown and the step 2 is run only one time for each plan. However, a more effective strategy is to put converters only in one/some (sparse conversion) nodes of the network; in this case the placement order of converters is the following: first, one converter is put on node 1; if the number of established connections is less than in the case of ubiquitous conversion with *Nc*=1, then another converter is put on node 2, and so on. In table 3 with sparse conversion, for *W*=2, *W*=3 and *W*=6 with only one converter in node 1 we will establish the same number of connections from *Nc*=1 in all nodes, and for *W*=4 and *W*=5 with 2 converters: one converter in node 1 and one converter in node 2 we will also establish the number of the connections from *Nc* =1 in all nodes. However, for *W*=4 and *W*=5, if we just one converter in node 1, we will establish 21 and 26 connections, respectively; so the total number of connections can not be established. Therefore the strategy proposed can be used to guide the placement of converters at the design of a network with RWA using the minimum number of converters, as seen in the figure 5.

Table 3. Lightpaths established x Wavelength

Number of lightpaths established									
W	No Conversion	Conversion	Sparse Conversion						
	Nc=0, for all	*Nc*=1,for all	Node	1	2	3	4	5	6
2	10	11	*Nc*	1	0	0	0	0	0
			11						
3	15	16	*Nc*	1	0	0	0	0	0
			16						
4	20	22	*Nc*	1	1	0	0	0	0
			22						
5	25	27	*Nc*	1	1	0	0	0	0
			27						
6	28	29	*Nc*	1	0	0	0	0	0
			29						

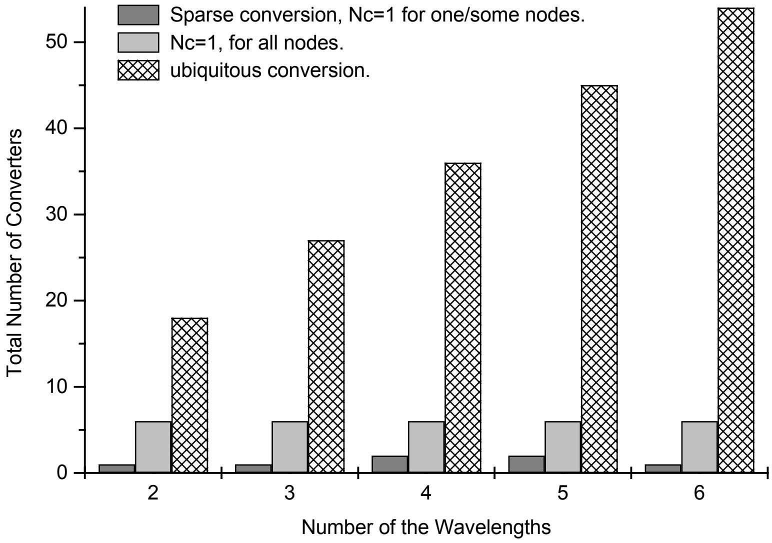


Figure. 5. Results for 6-node mesh Network. Total number of converters (Nt). Nt = ∑Nc for established connections from table III in function of wavelength plan. In ubiquitous conversion, W.δp, for each node.

3.1.2 Some comments

Integer Linear Programming models are popular in the literature as they provide formal descriptions of the problem. In practice, however, scalability to networks with at least 10´s of nodes, with 100´s of demands is required. In many cases all but trivial instances of theses ILP´s are computationally difficult with current state-of-the-art software. The complexity of our formulation grows as demonstrated in sub-section (2.1.4.2). In our 6-node mesh network, on average our strategy run be the optimization software CPLEX© [ILOG CPLEX 2003] took around five seconds on an Intel Pentium IV/1.6Ghz. However, in a large network as the National Science Foundation Network - NSFNET, with matrix of traffic given for [Swaminathan and Sivarajan 1998], and *Nc*=1 for all nodes, our strategy exceeded the CPU memory restriction. Thus, in next subsection a heuristic will be developed to find solutions to problems with size typically found in practice.

4. *K shortest path Heuristic* with limited number of converters (KSPNc)

Here we present a computationally less intensive heuristic algorithm based on *K* shortest paths for solving the Max-RWA problem with limited number of converters.

Assuming that in the lightpath request matrix, the largest lightpath request between any source-destination pair is *m*, we find the *K* shortest paths, wherein *K* is greater than *m*. The algorithm proceeds in following steps:

Step 1: *Fiding K shortest paths in terms of hop-length between all source-destination pairs in traffic matrix:*

The K shortest paths are store in *lightpaths*1, *lightpaths*2,…, *lightpathsK* arrays. Consider the first shortest path array i.e., *lightpaths*1 for processing and go to Step2.

Step 2: *Wavelength assignment to the lightpaths:*

For the lightpath which is not wavelength assigned in the traffic matrix, choose the path for that from the chosen K shortest path array. A typical lightpath between nodes (1) and (N) is represented as *node*[1], *node*[2],…,*node*[Q],…,node[N]; where nodes *node[2]*,…, *node[Q]* are nodes along the lightpath. The physical fiber links of the lightpath (*node*[1], *node*[2]) labeled as link 1, (*node*[2], *node*[3]) as link 2 and the last link (*node*[N-1], *node* [N]) as link N-1. The first physical link of the lightpath (here *node*[1], node[2]) is taken and scanned for a free wavelength. If a wavelength ς_j is free, then we try to find in all links of that lightpath for the availability of the wavelength ς_j. Then the algorithm proceeds further, differently for the following cases:

i. For the case of no conversion of wavelength along the lightpath, if the wavelength ς_j is available in all physical links along the lightpath, then we allocate that wavelength for the lighpath. If the continuity of the wavelength ς_j along the links in the ligthpath is not possible then, we scan for the next free wavelength ς_j on the link (*node*[1],*node*[2]). As before the availability of the wavelength ς_j on all the physical links on the ligthpat is checked, if the wavelength is available then it is assigned, else we scan for the next free wavelength on the link (*node*[1], *node*[2]) and the above procedure for wavelength assignment is repeated till the lightpath is wavelength assigned ot the wavelength in the link (*node*[1], *node*[2]) is exhausted.

ii. For the case of limited wavelength conversion, if wavelength ς_j is blocked on any link"*n*", then go back one physical link towards the source node of the lighpath "*n-1*" and try to obtain a free wavelength by wavelength conversion. If wavelength conversion is not available go back further one link "*n-2*" and try to obtain a free wavelength by wavelength conversion. Repeat the above procedure till a free wavelength is obtained or link 2 is reched on the lightpath. If a free wavelength is not available at link 2 then go to link 1 and choose a new free wavelength and traverse the physical links of the lightpath towards the destination node assigning wavelengths with or without conversion. While back tracking for a free wavelength if at any intermediate link if we get a free wavelength after conversion, then traverse from that link towards the destination node assigning wavelength with or without conversion. The wavelength conversion allowed at any node depends on the degree of conversion allowed and number of converters *Nc* allowed.

Step 3: Repeat the step 2 till all the lightpaths in the chosen array are exhausted.

Step 4: If any of the lightpaths in given traffic matrix is not wavelength assigned, then choose the next of the K shortest path arrays and go to step 2. If all the lightpaths are wavelength assigned or all the K shortest path arrays are exausted, then stop the algorithm.

4.1 Numerical Results

The NSFNET shown in figure 6 is a 14 node network with 21 edges. In this network each edge represents a pair of fibers, one in each direction.

The traffic matrix that has to be realized over the NSFNET is shown in the table V. We assumed that at most 3 multiple connections were permitted for a source-destination pair. The number of connections are chosen from 0,1,2,3 with equal probability for a source destination pair. The total number of connections to be set up is 268. For comparison with [Swaminathan and Sivarajan 2002], we assumed:

i. All nodes in the network are equipped with wavelength converters with limited conversion capability. Therefore, only the case “ii” in **step 2** from heuristic is considered**.**

ii. The degree of conversion is 3. For help the reader, see fig.1b, it the degree of conversion is 2.

First, we relax the integer constraints of mathematical formulation, as [Swaminathan and Sivarajan 2002]. For a given wavelength, we find LP upper bound.

After, we executed the KPSH and KPSNc algorithm on the NSFNET network with the value of K=5. The greater the value of K, more will be number of connections realized, because there are more alternative paths available for wavelength assignment. The KPSNc results based on the number of converters are captured in table along with results of LP (upper bound) and KPSH Heuristic with large Nc from [Swaminathan and Sivarajan 2002].

Comparing in table 5, we find that in terms of performance (Established Connections); LP (upper bound), KPSH, KPSNc with Nc=7 and KPSNc with Nc=5 gives better performance in that order. In LP, this happens because of the relaxed integer constraints. In KSPH, this happens because of large number of converters. The KSPNc had limited number of converters, therefore the performance is less or the same from W=10 to W=20. However, in the network practical design the cost is minimized when the minimum number of converters is used. Then, KSPNc is a heuristic cost-effective.

5. Conclusions

In this paper, we presented ILP formulation and a *K-shortest path heuristic* approach for the limited number of converters and kinds of conversion in optical networks.

The formulation/strategy proposed in this paper have a significant impact to the well known RWA problem. First, it can help understand the relationship between the number of wavelengths required and the number of converters. Second, it can be used to guide the placement of converters at the design of a network. Third, a new feature of the proposed formulation is that any kind of conversion can be made in each node of the network. This is obtained by the more general constraints (10) and (11).

By recently developed heuristic technique advancements [Koster and Zymolka 2005], the best known solution could be improved for more instances with minimum converter wavelength assignment, again proving optimatily. Besides further founding the benefit of our approach, this observation also indicates that the heuristic algorithm are still improvable.

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Table 4. Session Matrix for NSFNET (268 connections)

0	1	3	1	1	1	3	0	2	0	1	2	0	3
0	0	0	2	2	2	1	1	1	2	1	0	1	3
3	2	0	3	0	1	2	3	1	3	1	2	2	0
3	1	0	0	1	1	2	3	2	2	1	2	1	3
1	3	0	2	0	1	0	2	0	3	0	1	1	3
1	2	1	3	2	0	1	3	3	1	0	1	1	2
2	2	3	1	3	3	0	0	3	1	2	0	3	3
3	1	2	3	1	0	1	0	0	3	2	0	3	0
3	0	1	3	3	3	1	0	0	2	1	1	1	0
0	0	0	1	2	0	2	0	1	0	1	0	0	3
1	0	0	2	0	3	0	1	0	3	0	3	1	3
2	3	1	1	3	2	3	2	2	2	2	0	1	3
2	0	1	2	0	1	2	0	3	0	2	11	0	3
1	1	0	2	1	0	1	3	0	1	2	1	3	0

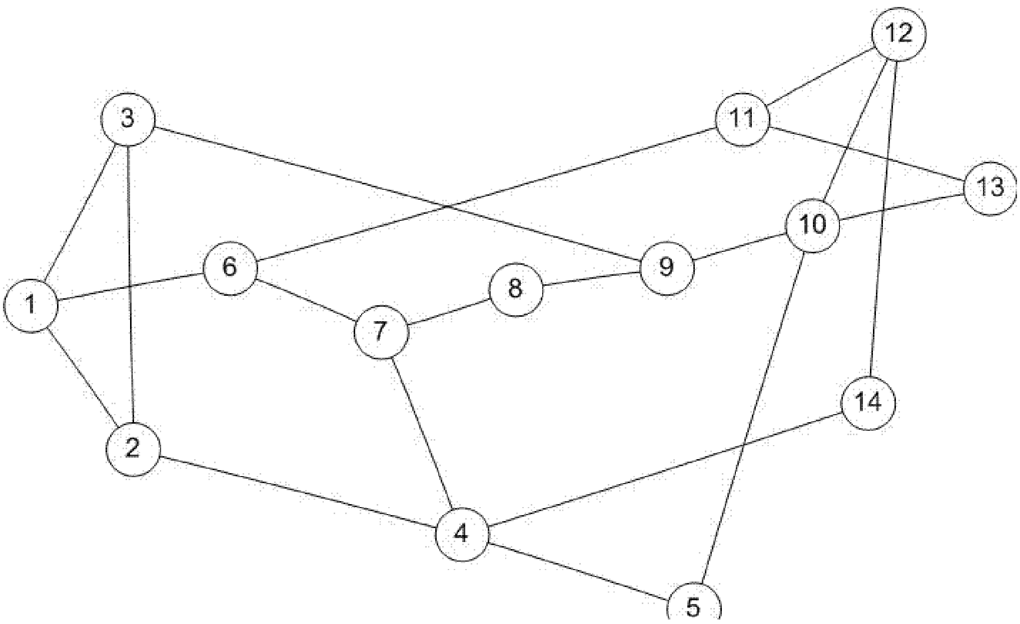

Figure 6. NSFNET

Table 5. Result obtained for NSFNET by LP formulation and heuristics

W	Established Connections			
	Upper Bound (LP)	Heuristic		
		KSPH	KSPNc	
		Nc=W.δ_p	Nc=7	Nc=5
10	198	187	187	182
11	208	196	196	191
12	218	209	207	203
13	228	220	218	214
14	238	229	227	224
15	248	238	236	233
16	258	246	243	239
17	263	252	247	247
18	267	255	252	251
19	268	258	256	258
20	268	262	259	259
21	268	264	261	260
22	268	266	265	264
23	268	267	267	267
24	268	268	268	268
25	268	268	268	268
26	268	268	268	268