

Connectivity in a Multi-radio, Multi-channel Heterogeneous Ad Hoc Network

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***Abstract.** Future wireless networks are expected to integrate heterogeneous devices equipped with multiple radios and different characteristics. Nodes equipped with a single communication interface will co-exist with nodes equipped with multiple radios that can transmit and receive simultaneously. Therefore, it is important to understand how these heterogeneous radios can affect connectivity and overall multihop network performance. Maintaining connectivity in ad hoc networks has been a major challenge and many complex topology control algorithms have been investigated. In this paper, we study the connectivity of a heterogeneous ad hoc network. We consider two types of nodes: nodes equipped with a single communication interface with communication range r , and Dual-Mode (DM) nodes, equipped with two communication interfaces with two different communication ranges r and r_f , respectively, where $r_f > r$. We assume the radios in the DM nodes operate in two different channels. We provide a theoretical analysis of connectivity in a linear network and we present simulation results for the two-dimensional case that show the impact of DM nodes on connectivity, broadcast latency and robustness in static and mobile scenarios.*

1. Introduction

The design of ad hoc and sensor networks has become an attractive research topic due to the several possible applications ranging from hostile environmental monitoring to smart buildings. In almost all applications, the multihop routing functionality plays a fundamental role in the network performance. Therefore, keeping the network connected is a basic requirement. Connectivity in a multihop network can be defined as the ability to provide direct or multihop communication between any pair of nodes in the network. Connectivity issues in homogenous ad hoc networks have been extensively studied in the literature [5][6][7][8][13][17][18][21]. The basic problem is how to select the nodes transmission power to achieve connectivity, while optimizing the network capacity and reducing power consumption. Several topology control mechanisms have been proposed to ensure connectivity in ad hoc and sensor networks [11][12][15][16].

Recently, nodes equipped with multiple radios have been considered as a promising way to improve the capacity of wireless networks [3][4]. As pointed out in [4], it is not unrealistic to consider that future devices will be equipped with radios operating on different frequency bands, such as 802.11a and 802.11b/g that have different bandwidth, range and fading characteristics. With multiple interfaces a node can transmit and receive simultaneously improving the relay capacity, connectivity and the spectrum utilization in an ad hoc network. However, most theoretical analyses of

connectivity in ad hoc networks have considered nodes with a single radio with homogeneous characteristics [5][6][8][13][17]. In [2], the author considers an inhomogeneous range assignment to ensure connectivity, but connectivity is estimated in [2] as a function of the isolation probability using a relation that is valid only for highly dense networks.

In this work we address the connectivity problem in a wireless ad hoc network with single-radio and multi-radio nodes, called Dual Mode (DM) nodes. Our basic goal is to show how the DM nodes can enhance the network connectivity, improve robustness and reduce the power consumption of the overall system. Each DM node is equipped with two communication interfaces with two different communication ranges r and r_f , respectively, where $r_f > r$. We assume that longer-range radios interfaces (r_f) operate in a different frequency band from the short-range radios (r). We provide a theoretical study of connectivity in one-dimensional network with DM nodes and present simulation results for the two-dimensional case to show the effects of the number of DM nodes and the longer range communication range on the network connectivity, broadcast efficiency and robustness. Additionally, we analyze the effects of mobility on the network performance when the DM nodes move according to the Random Way Point model.

In our analytical model, nodes positions are distributed according to a Poisson process of intensity λ on an infinite line, and a fraction f of the nodes are DM nodes. The network can be represented as a variation of the Poisson Boolean Model [14] and for each realization of the model we can associate a random graph $G(\lambda, f, r, r_f)$ by defining the points of the Poisson Boolean Model as nodes and placing edges between nodes that can directly communicate. Then, we analyze the connectivity in $G(\lambda, f, r, r_f)$ by computing the distribution of the probability that two arbitrary nodes A and B, whose distance $d(A,B)=x$, are connected to each other, which is denoted by $P_c(x)$.

Since it is not feasible to extend the analytical model to the two dimensional case, we perform a simulation analysis, where we measure connectivity as the fraction of nodes that receives a broadcast message initiated by a randomly selected node. Differently from the results presented in [2][17] that consider only the simplistic fixed radius graph model of the network, we include the effects of MAC and physical layers. Our primary goal is to show that the DM nodes represent a supplementary alternative to improve network connectivity, and other aspects such as capacity, reliability and network lifetime, without requiring any complex topology control mechanism.

The remainder of this paper is organized as follows: In Section 2, we discuss some related work and the main contributions of our work. Next, in Section 3, we describe our one-dimensional model for a network with DM nodes and find the distribution of the probability that any two nodes are connected as a function of the distance between them. In Section 4, we present simulation results that illustrate the impact of the DM nodes on connectivity, power requirements, broadcast latency and fault tolerance. In Section 5, we consider mobility in the performance analysis. Finally, we present some concluding remarks and future work in Section 6.

2. Related Work

Several papers have addressed the connectivity issue in multihop wireless networks by proposing topology control mechanisms (see [11][12][15][16] and references therein). Theoretical analyses of connectivity are often used to support any new topology control mechanism. In [8], the authors explored a phase transition represented by a critical transmission power level for all nodes beyond which the network becomes connected. Simulation results in [13] showed that there is a critical

threshold for the communication radius above which the network is connected with a high probability. In [17], the authors provide tight upper and lower bounds on the critical communication range for connectivity for one-dimensional homogeneous networks and nontight bounds for two and three-dimensional cases. Closed analytical expressions are presented for the one-dimensional case, while simulations are used for two and three dimensional cases. A mobile scenario is also considered, in which the authors study the communication range required to ensure connectivity for a given fraction of the time. In [18], the authors used a grid topology to show necessary and sufficient conditions related to the nodes transmit power to ensure coverage and connectivity. In [6], the authors analyzed the connectivity of a homogeneous ad hoc network by taking into account the interference between nodes in the construction of the network graph.

Most previous works consider homogeneous ad hoc networks in which the nodes have fixed communication range. However, the availability of WLAN interfaces with different transmission rates and communication ranges (e.g. 802.11a/b/g) has motivated the study of ad hoc networks where nodes are equipped with multiple radios. The authors in [4] performed real experiments with nodes equipped with two 802.11 radios (different combinations of 802.11a/b/g) and analyzed the performance of a new routing protocol for this heterogeneous ad hoc network. The results in [4] showed that dual-radio nodes can be exploited to improve the network throughput.

To the best of our knowledge, no other work provides a detailed analysis of connectivity in a heterogeneous ad hoc network with dual-radios. The authors in [5] studied the connectivity of homogeneous and hybrid ad hoc networks, but they consider only single-radio nodes and the hybrid network are formed by wireless nodes and fixed Base Stations connected to each other through wired links. In the one-dimensional case, the authors computed the distribution of the probability that two arbitrary separated by a distance x are connected to each other ($P_c(x)$) in a large scale homogeneous ad hoc network and showed that the network is almost surely disconnected since $P_c(x)$ is bounded and goes to zero as $x \rightarrow \infty$. Then, they showed by simulations and real experiments, for both one and two dimensional cases, that connectivity can be improved in by introducing fixed Base Stations connected to each other through wired links. In this work, we compute the distribution of $P_c(x)$ for a linear heterogeneous ad hoc network with Dual-Mode (DM) nodes and provide simulation results for the two dimensional case that shows the impact of the Dual-Mode nodes on the network performance.

In another related work [2], the author considers single-radio nodes, but with inhomogeneous communication ranges and approximates k -connectivity probability $P(k\text{-con})$ as the probability the minimum node's degree is greater or equal to k ($P(d_{min} \geq k)$). However, this approximation is only valid for probabilities close to 1, which is not the case in sparse ad hoc networks where to assure high connectivity is an important issue. Also, the simulation results in [2] use a simple fixed radius graph model.

Our main goal in this work is to study the connectivity in heterogeneous ad hoc networks, and in particular, we are interested in weakly connected topologies, which is a much more critical scenario than the one considered in [2]. For large scale applications, such as sensor networks, connectivity may decrease with the time, as nodes "die" due to energy constraints. Also, in order to improve the network life time, even under high nodes density, it is desirable to operate with the minimal number of nodes to ensure coverage and connectivity. Differently from [2], we do not assume high nodes density and we consider both single-radio and dual-radio (Dual-Mode) nodes. Thus, our one dimensional model allows the study of connectivity as a function of the nodes density,

i.e., it can be applied in sparse as well as in dense networking scenarios. In addition, our simulation analysis of the two dimensional case includes the effects of the physical and MAC layers, which is not considered in [2], and we also show the impact of the dual mode radios in other aspects such as robustness and broadcast efficiency.

3. One-Dimensional Model

We consider a wireless ad hoc network in which a fraction f of the nodes are equipped with two communication interfaces (DM nodes) with two different communication ranges r and r_f , respectively, where $r_f > r$. Thus, two arbitrary nodes A and B at distance $d(A,B)$, are said to be directly connected if $d(A,B) < r$, or if $d(A,B) < r_f$ and both nodes (A and B) are Dual-Mode nodes.

A comparison between a network with 4 typical nodes and a network with Dual-Mode nodes is presented in Figure 1. The simple example in Figure 1 shows that the network graph becomes connected when B and D are Dual-Mode nodes, since they can directly communicate using the longer range radio. Our problem is to find the distribution of $P_c(x)$ as a function of λ , r , r_f and f , and we have to consider three different cases:

Case I: $0 \leq x < r$. This is the trivial case and $P_c(x) = 1$.

Case II: $r \leq x < r_f$. In this and in the next case, the connectivity between two arbitrary nodes A and B depends on the following events:

- $E1$: A and B is not a DM node;
- $E2$: A and B are DM nodes;
- $E3$: Either A or B is a DM node;
- $E4$: There is a relay node K between A and B, where K is a node directly connected to B and connected to A through one or more hops [5].

Given above set of events, $P_c(x)$ for Case II can be expressed as

$$P_c(x) = \Pr\{E4|E1\}\Pr\{E1\} + \Pr\{E4|E3\}\Pr\{E3\} + \Pr\{E2\}. \quad (1)$$

Case III: $x \geq r_f$. In this case, the two nodes can only communicate through multiple hops, i.e., $P_c(x) = \Pr\{E4\}$. Next, we calculate $P_c(x)$ for cases II and III. The general solution for $P_c(x)$ is extremely complex and we consider the particular case when $r_f = 2r$.

3.1 Case II: $r \leq x < r_f$

We assume that the DM nodes are randomly selected between all nodes in the network such that we have $\Pr\{E2\} = f^2$, $\Pr\{E1\} = (1-f)^2$ and $\Pr\{E3\} = 2(1-f)f$. Now, consider the event E4 in which $d(K,B) = \xi$, and $0 \leq \xi < x$. Since the nodes are distributed according to a Poisson process, the distance between two nodes is an exponential random variable and we have

$$\Pr\{E_4 | E_1\} \Pr\{E_1\} = (1-f)^2 \int_0^r P_c(x-\xi) \lambda e^{-\lambda \xi} d\xi, \quad (2)$$

$$\Pr\{E_4 | E_3\} \Pr\{E_3\} = (1-f)f \int_0^r P_c(x-\xi) \lambda e^{-\lambda \xi} d\xi + (1-f)f \left[\int_0^r P_c(x-\xi) \lambda e^{-\lambda \xi} d\xi + f \int_r^{r_f} P_c(x-\xi) \lambda e^{-\lambda \xi} d\xi \right]. \quad (3)$$

Substituting the correspondent probabilities in (1) and rearranging the expression we obtain

$$P_c(x) = (1-f^2) \int_0^r P_c(x-\xi) \lambda e^{-\lambda\xi} d\xi + (1-f) f^2 [e^{-\lambda r} - e^{-\lambda r_f}] + f^2. \quad (4)$$

Using a transformation of variables and the Leibnitz rule to differentiate integrals, we obtain the following differential difference equation:

$$\frac{dP_c(x)}{dx} + aP_c(x) + bP_c(x-r) = c \quad (5)$$

where $\alpha = \lambda f^2$, $b = (1-f^2)\lambda e^{-\lambda r}$ and $c = \lambda f^2 [1 + (1-f)(e^{-\lambda r} - e^{-\lambda r_f})]$. Then, our problem is to solve (5), and making $x=r$ in (4), we can obtain the initial condition $P_c(r) = 1 - (1-f^2)e^{-\lambda r}$.

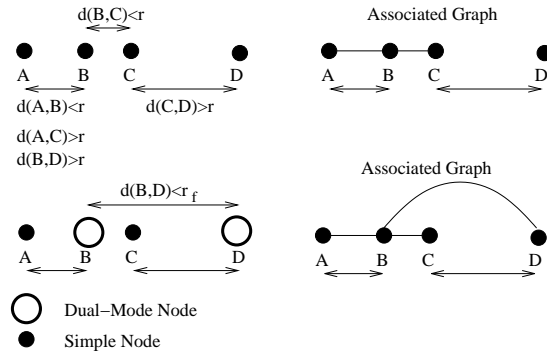


Figure 1: Network with Dual-Mode nodes and the associated network graph.

3.2 Case III: $x \geq r_f$

Note that, when $x \geq r_f$ we can rewrite

$$P_c(x) = \Pr\{E4|E1\}\Pr\{E1\} + \Pr\{E4|E2\}\Pr\{E2\} + \Pr\{E4|E3\}\Pr\{E3\}. \quad (6)$$

$\Pr\{E4|E1\}\Pr\{E1\}$ and $\Pr\{E4|E3\}$ are given by (2) and (3), respectively, and $\Pr\{E4|E2\}$ can be obtained considering two possibilities:

i) $d(K,B) < r$, which means that K does not need to be a DM node to be the next hop from B to A.

ii) $r \leq d(K,B) < r_f$, in this case, K has also to be a DM node. Then, after some manipulations we find

$$\Pr\{E4 | E2\} \Pr\{E2\} = f^2 [\Pr\{i\} + f \Pr\{ii\}] = f^2 \int_0^r P_c(x-\xi) \lambda e^{-\lambda\xi} d\xi + f^3 \int_r^{r_f} P_c(x-\xi) \lambda e^{-\lambda\xi} d\xi \quad (7)$$

Combining (2), (3), (6) and (7), we have

$$P_c(x) = \int_0^r P_c(x-\xi) \lambda e^{-\lambda\xi} d\xi + f^2 \int_r^{r_f} P_c(x-\xi) \lambda e^{-\lambda\xi} d\xi \quad (8)$$

Using again a simple transformation of variables and differentiating (8) we obtain

$$\frac{dP_c(x)}{dx} + \lambda(1-f^2)e^{-\lambda r} P_c(x-r) + f^2 \lambda e^{-\lambda r_f} P_c(x-r_f) = 0, \quad (9)$$

which is also a differential difference equation, as in case II, and the initial condition is $P_c(r_f)$. For a homogeneous network, i.e., $f=0$, (9) is reduced to the same expression obtained in [5]. Also, for $f=1$, Eq.(9) results in the same expression for a homogeneous network with r taken twice as big ($2r$).

The general solution for $P_c(x)$ is relatively complex. Next, we solve for the particular case when $r_f=2r$ using the Laplace transform technique described in [1]. The solution for case II can be easily obtained and is given by:

$$P_c(x) = \frac{c-b}{a} + \left(P_c(r) - \frac{c-b}{a} \right) e^{-a(x-r)} \quad (10)$$

Note that (10) is valid for $a>0$. Using (8) and the solution for case II (Eq. 10), we can compute the initial condition for case III. Then, for $r_f=2r$, we have

$$P_c(r_f) = \frac{c-b}{a} (1 - e^{-\lambda(r_f-r)}) + \left(\frac{aP_c(r) - c + b}{f^2} \right) \frac{(e^{-\lambda(r_f-r)} - e^{-a(r_f-r)})}{(1-f^2)} + f^2 (e^{-\lambda r} - e^{-\lambda r_f}) \quad (11)$$

for $0 < f < 1$ and $\lambda > 0$. After solving (9) with the above initial condition we get $P_c(x)$ when $x \geq r_f=2r$:

$$P_c(x) = P_c(r_f) \left[1 + \sum_{n=1}^{\lfloor \frac{x}{r} \rfloor} (-1)^n (b + a e^{-\lambda f})^n \left(\frac{(x - rn)^n - (2r - rn)^n}{n!} \right) \right] \quad (12)$$

Next, we analyze the impact of the number of DM nodes on the network connectivity for the homogeneous and heterogeneous case. Note that our solution for $P_c(x)$ is valid only for $0 < f < 1$. Thus, we obtain $P_c(x)$ for the homogeneous case ($f=0$), by solving (5) and (9) with $f=0$, which matches the solution in [5],

$$P_c(x, f=0) = \sum_{i=0}^{\lfloor x/r \rfloor} \frac{(-\lambda e^{-\lambda r} (x - ir))^i}{i!} - e^{-\lambda r} \sum_{i=0}^{\lfloor x/r \rfloor - 1} \frac{(-\lambda e^{-\lambda r} (x - (i+1)r))^i}{i!}, \text{ for } x \geq r. \quad (13)$$

Without loss of generality assume $r=1$ and consider two nodes A and B separated by a fixed distance, say $x=10r$. Then, $P_c(x=10r)$ gives the probability that there exists a connected component in the network associated graph that spans the distance x and connects A to B. The connectivity probability varies with the density λ as shown in Figure 2(a) and there is a critical density λ_c above which the connectivity abruptly approaches one for both the homogeneous ($f=0$) and the heterogeneous networks with different f values. This behavior has been observed for homogeneous ad hoc networks and our results confirm that it is also the case in heterogeneous multihop networks with dual mode radios. Further, we can observe a certain connectivity improvement as f increases. However, the improvement depends on f and λ . For small densities, i.e., $\lambda < \lambda_c$, the connectivity probability is small (less than 0.4) even having almost all nodes with two radios ($f=0.999$). The main improvement is achieved in the transition phase ($1 < \lambda < 5$). But only high fractions of DM nodes can provide significant improvement. As shown Figure 2(a), the curves for $f=0.1, 0.3$ and 0.5 are very close to the curve for $f=0$, and only $f=0.8$ and 0.999 results in high connectivity gain for $1 < \lambda < 3$.

We further analyze the impact of the DM nodes by plotting $P_c(x)$ as a function of x for three nodes density scenarios: low ($\lambda=0.4$), medium ($\lambda=2$) and high ($\lambda=5$) in Figure 2(b), Figure 3(a) and (b), respectively. A clear improvement can be seen for $\lambda=0.4$ and $\lambda=2$ with $f=0.5$ and 0.99 , but it is not enough to change the decreasing behavior of $P_c(x)$, which means that a large-scale heterogeneous

network is almost surely disconnected under these density conditions. For $\lambda=5$, $P_c(x)$ is not significantly affected with $f=0.1$ and $f=0.5$. On the other hand, $P_c(x)$ approaches one with $f=0.99$, which is a situation in which almost all nodes have a higher communication range ($r_f=2r$).

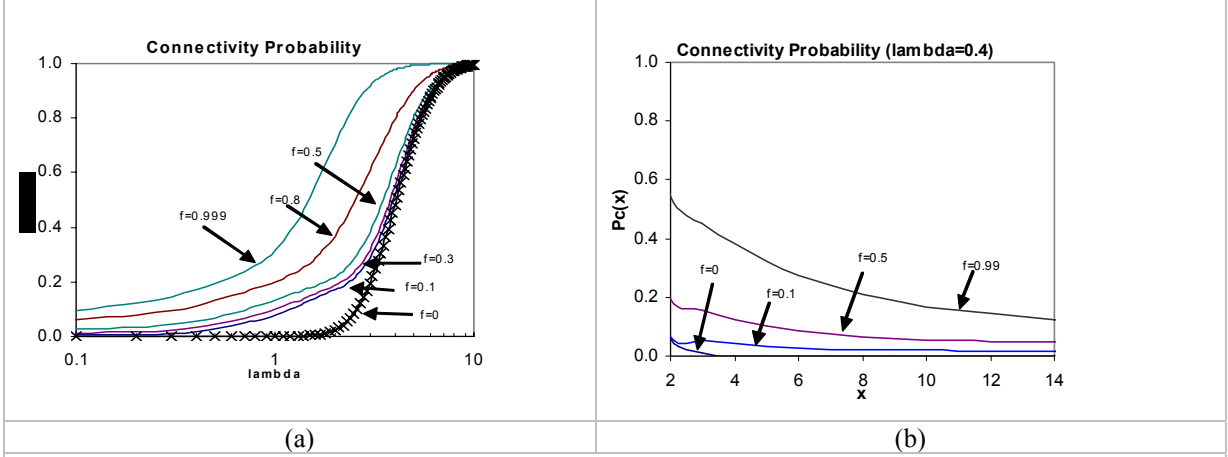


Figure 2: (a) Connectivity Prob. for ($r_f=2r$ and $x=10$), (b) Connectivity Prob. vs. x ($\lambda=0.4$)

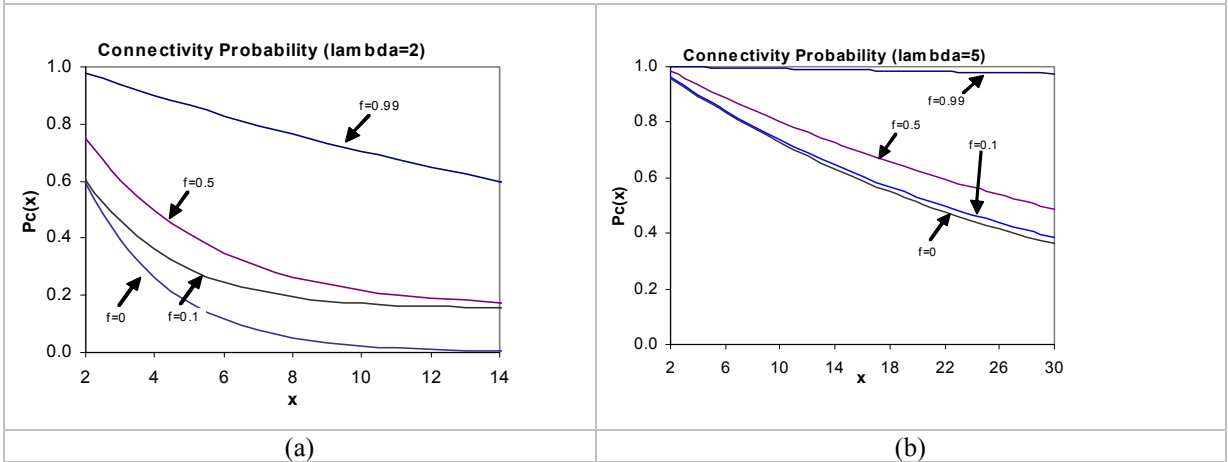


Figure 3: (a) Connectivity Prob. vs x ($\lambda=2$), (b) Connectivity Prob. vs. x ($\lambda=5$)

In summary, the one-dimensional model shows that the DM nodes increase the connectivity compared to the homogeneous case, but they have only a limited effect, since the network is almost surely disconnected as $P_c(x)$ monotonically goes to zero for very large x . High connectivity can only be achieved when almost all nodes have two radios and under high density of nodes. Any further increase in connectivity is possible only if we can increase the range r_f and/or r . In the next section we present simulation experiments for the two dimensional case for different r_f/r relations.

4. Connectivity of a Two-Dimensional Heterogeneous Ad hoc Network

In the two dimensional case, a closed form for $P_c(x)$ in a homogeneous network is not feasible to obtain [5], and the problem becomes even more complex in the heterogeneous case. Hence, our goal here is to show by simulations the impact of the DM nodes on the network performance. If DM nodes are able to significantly improve connectivity, complex topology control algorithms may not be required for heterogeneous ad hoc networks.

In simulation analysis, the connectivity is usually estimated by computing the normalized size of the largest cluster of connected nodes in the network, which is known as the percolation probability [5]. Most related papers measure connectivity on the graph constructed by the fixed radius model, where the nodes are distributed over a given area and nodes can directly communicate with other nodes inside their fixed transmission range [2][5][17]. However, in practice, the fact that the distance between two nodes is smaller than their transmission range does not assure that a transmission between them will be successful, since the packet can be lost due to failures in different layers of the protocol stack. Therefore, it is import to measure connectivity at high layers, such that we can capture the system performance from the application/user perspective. Here, we measure the connectivity by computing the fraction of nodes that received a broadcast packet sent by a randomly chosen source node. We denote T_b as the period of time between two consecutive broadcasts initiated by the source node.

We have used NS version 2.26 as the simulation environment, and the nodes are uniformly distributed in a 1000m by 1000m square area. We have simulated 100, 250 and 350 nodes and all results are averaged over twenty runs. We have created enhanced NS Mobile Node module, with dual interface capability. In the modified protocol stack, each interface of DM mobile nodes consists of a separate link layer, an ARP module, interface priority queue, IEEE 802.11 MAC and Physical layers. When the routing layer of a DM node receives a broadcast packet, it forwards the packet to both the interfaces. Furthermore, we have enhanced the physical layer implementation of NS in order to take into account the thermal noise and all ongoing transmissions in the channel to compute the *SINR* (Signal to Interference and Noise Ratio) at a given receiver, which is used to detect whether a packet can be successfully received. Table I provides different parameters used in the simulation. We consider only broadcast traffic and, initially, we compare the connectivity with and without DM nodes in a static network. Next, we analyze the broadcast latency and the network’s robustness. Finally, we present some results in a mobile scenario.

Table I: Simulation Parameters

Parameters	Value
Number of Nodes	100, 250, 350
Simulation Time	200s
Broadcast period (T_b)	50s
Area	1000m x 1000 m
Transmission rate	1Mbps
MAC Protocol	IEEE 802.11
Propagation Model	Two Ray Ground with cross-over distance 80 m

Figure 4(a) shows the connectivity of a 250 simple nodes network as a function of the transmission radius measured at the network layer in our simulation environment (NL) and using a simple fixed radius graph model. As expected, we can observe an abrupt transition in the connectivity level as the transmission range increases in both cases. Almost no difference between the models is perceived when the network is totally disconnected ($r < 50$ m) or when it is connected ($r > 100$ m), but in the transition, the NL model resulted is a smaller connectivity level than the graph model. Also, the higher variability in the transition phase (see the error bars for a 95% confidence interval) shown in Figure 4(a) suggests that, when the network is in this critical phase, results obtained with a simple graph model may not be completely true in practice. MAC layer collision and wireless channel characteristics are among the dominant factors which influences connectivity. In all the remaining

results, relatively variability was observed for low connectivity levels, as in Figure 4(a), while for high connectivity values ($> 90\%$), the errors are very small.

Figure 4(b) shows the increase in connectivity when DM nodes are added in the network. We have set the smaller transmission range to $r = 70$ m, which is in the critical transition region (see Figure 4(a)), and we considered different values of the higher transmission range r_f . A considerably connectivity gain is achieved as the fraction f of DM nodes increases. For small values of f , the network remains disconnected regardless of used r_f . On the other hand, for $f > 0.2$ all values of $r_f > 2r$ resulted in an almost connected network. From this point on, there is not much connectivity gain by adding more DM nodes, which means that there is an upper bound on the number of DM nodes that increase connectivity. We can also notice that when r_f increases from $3r$ to $5r$, the connectivity gain is higher than when it increases from $5r$ to $8r$. So, we can say that r_f values larger than a certain threshold does not help to maximize connectivity.

Figure 5(a) depicts the connectivity for a network with 350 nodes. As can be seen, a significant connectivity improvement can be obtained with a smaller number of DM nodes compared to the network with 250 nodes. In this case, more than 97% of the nodes are connected with $f=0.1$ and for all r_f values greater than $2r$. In Figure 5(b), we plot the fraction of connected nodes as a function of f with $r_f=3r$ for different number of nodes. The results suggest that as the nodes density approaches the critical value and a smaller number of DM nodes can result in a greater connectivity gain.

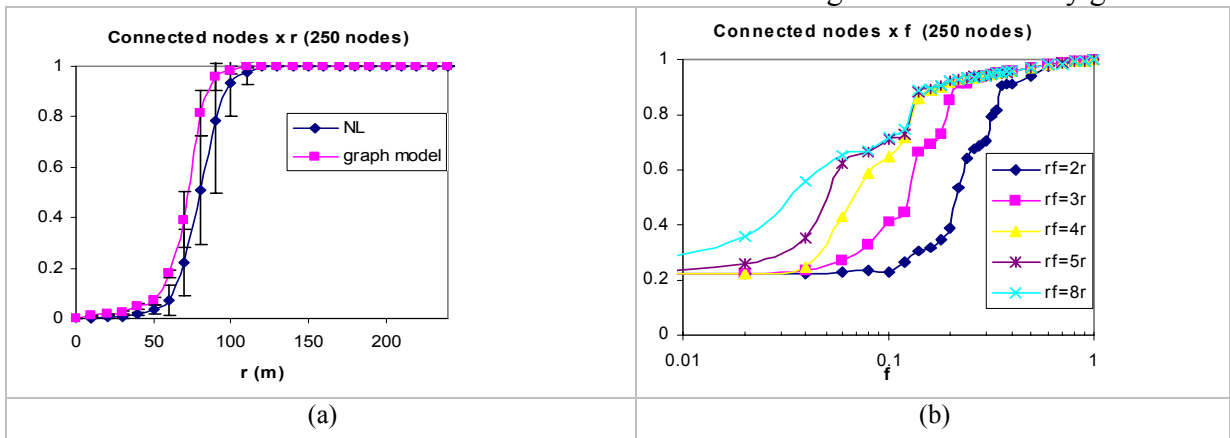


Figure 4: (a) Connectivity in a homogeneous network. (b) Connectivity in a heterogeneous network as a function of the fraction of DM nodes.

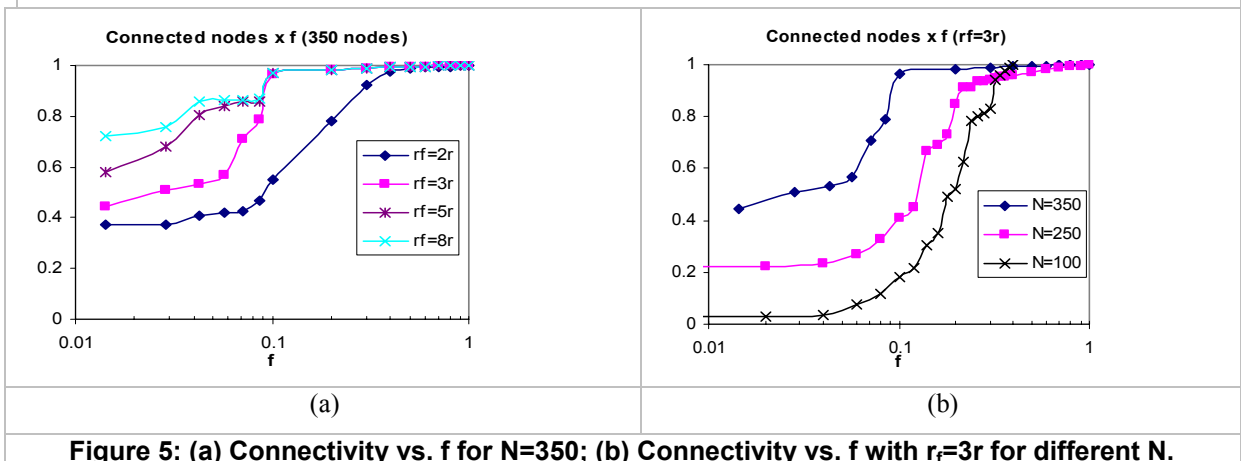


Figure 5: (a) Connectivity vs. f for $N=350$; (b) Connectivity vs. f with $r_f=3r$ for different N .

4.1 Optimal f and Transmitting Power for High Connectivity

The above results suggest that there exists an optimal fraction of DM nodes (f_0) depending on λ and on the relation r_f/r that can transform a disconnected network into an almost surely connected network. In Table II, we show f_0 and the maximum transmitting power used in the smaller range interface (r) required to achieve 95% (+/-2%) of connectivity for different r_f/r relations. The relation between the maximum transmitting power and communication range was obtained using the two ray ground path loss model. According to Table II, in homogeneous networks, each node has to transmit at 10.5mW and 7.21 mW, which gives communication ranges $r=110$ m and $r=100$, for 250 and 350 nodes respectively. In the heterogeneous case, we have fixed $r=70$ m by setting the maximum transmitting power in the corresponding to 2.62 mW and increased r_f . As can be seen, f_0 decreases as the number of nodes increases. Also, for a given number of nodes (density), fewer DM nodes can result in high connectivity as r_f increases. However, it seems that there is also a maximum r_f after which no improvement is achieved. With 250 nodes, the reduction in f_0 is only 0.04 as we increase r_f from $3r$ to $5r$, and with 350 no reduction is observed.

Table II – f_0 and maximum transmitting power in interface r for 95% connectivity

		<i>Heterogeneous Net.</i>		<i>Homogeneous Net.</i>
		f_0	<i>Tx Power on interface (r)</i>	<i>Tx Power (r)</i>
$N=250$	$r_f=2r$	0.5	2.62 (70 m)	10.56 (110 m)
	$r_f=3r$	0.26		
	$r_f=5r$	0.22		
$N=350$	$r_f=2r$	0.4	2.62 (70 m)	7.21 (100 m)
	$r_f=3r$	0.1		
	$r_f=5r$	0.1		

If we consider the maximum transmitting power at the interface r , 75% and 64% reduction can be obtained with DM nodes to maintain the same connectivity compared to a homogeneous network with 250 and 350 nodes, respectively. It is important to remark that although a reduction in the simple nodes transmitting power can be obtained, the DM nodes still need higher power to transmit at longer distances. But a relatively small fraction of DM nodes (e.g. 10% with 350 nodes) is still enough to maintain high connectivity. This idea can be exploited in sensor networks, for example, as the introduction of special devices with multiple interfaces can reduce the sensors' transmitting power required to maintain high connectivity. The heterogeneous network can also be an alternative to topology control algorithms that adjust the transmitting power to ensure connectivity.

4.2 Broadcast Latency and Robustness

We define the broadcast latency as the time needed for a packet to travel from source to all connected nodes in the network. This is an important parameter in route discovery process and in some critical application, such as environmental monitoring with sensors. In Figure 6(a), we plot the broadcast latency for 250 and 350 nodes. We assume $r_f=3r$ and compare two cases: $f=0$ and $f=0.1$. As we can see, the broadcast latency is smaller in the heterogeneous network. Indeed, 35% and 23% reduction is observed for $N=250$ and 350, respectively. In fact, the DM nodes not only increase the connectivity, but also increase the speed the information propagates through the network as the longer range links contributes to reduce the average hop distance between nodes.

We have also analyzed the broadcast in a network where a fraction p of the nodes are randomly switched off to simulate energy constraints. We call p as the turn off probability. By measuring the network connectivity as a function of p we have information about the network robustness or fault tolerance, i.e., the ability of the network to remain connected as nodes turn off their radios. The results in Figure 6(b) confirm the connectivity improvement with the DM nodes. However, as p increases, the number of nodes that received the broadcast monotonically approaches to zero.

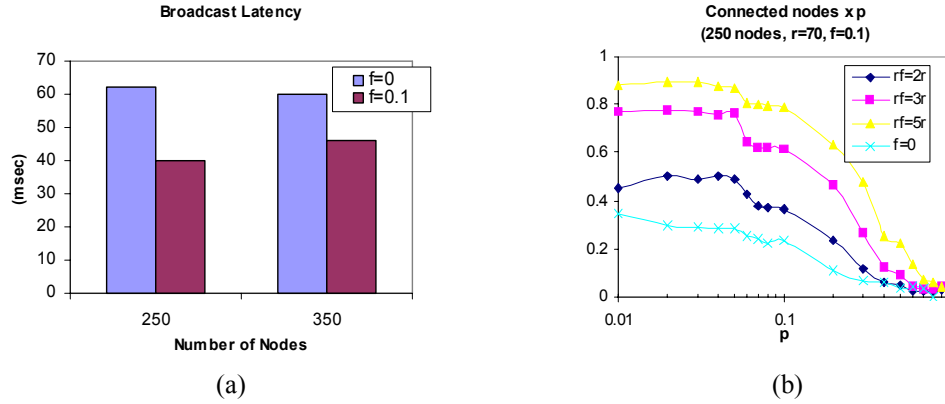


Figure 6: (a) Broadcast Latency in a homogeneous network ($f=0$) and with DM nodes ($f=0.1$ and $rf=3r$). (b) Connectivity vs. turn off probability.

5. Connectivity with Mobile Nodes

It is known that the capacity of a static ad hoc network is interference limited and per node throughput decreases with the network size [8]. In [10], Grossglauber and Tse showed that the long-term per node throughput in a mobile ad hoc network can be kept constant as the network scales by exploiting nodes mobility. The idea in [10] is to split the packets of each source node to as many nodes as possible, such that these intermediary nodes would serve as relay nodes and whenever they get close to the final destination, they hand the packets off to the destination. However, the splitting process of the packets among different nodes increases end-to-end delay to deliver the complete message. On the other hand, mobility has a negative effect on the multihop routing; as it introduces frequent topology changes that require routing table updates and more control overhead. In applications like sensor networks, where nodes usually have limited mobility, the overhead caused by topology changes due to mobility is reduced, but we can exploit the longer communication range of Dual-Mode nodes and the mobility effect in this scenario to achieve better connected topologies.

As we showed above, the DM nodes considerably improve the network connectivity. Although the average improvement was obtained by randomly placing the DM nodes, the overall connectivity gain can be limited if the DM nodes are clustered in a small area. In fact, since all nodes are randomly placed, disconnected nodes or clusters can always exist. Our goal here is to analyze the effects of allowing the DN nodes to move into the covered area. It is important to note that the connectivity improvement achieved with mobile multi-radio nodes is not constant, i.e., under mobility, the network connectivity changes with the time. Furthermore, the connectivity can vary randomly, depending of the mobility characteristics of each node.

Initially, we consider all nodes moving according to the Random Way Point (RWP) model. The main mobility parameters defined in the RWP model are shown in Table III. Recall that we measure connectivity as the fraction of nodes that received a broadcast packet initiated by a random selected node, thus to capture the effect of mobility, we also have to consider the period of time between two consecutive broadcasts T_b . If T_b is high compared with the mobility level of the DM nodes, the mobility effect may not be sufficiently exploited. For example, a DM node can pass through a cluster of isolated nodes during the period where no packets are being transmitted, and no improvement in connectivity is experienced. We simulated a 100 nodes with $r_f=3r$, $T_b=50s$, pause time $T_p=20s$ and increased f . As depicted in Figure 7(a), in a mobile environment, the impact of the DM nodes on connectivity is even greater than in a static scenario. As we discussed earlier, for the static scenario with 100 nodes, the DM nodes are not effective to increase the connectivity because the nodes density is considerably below the critical value. When mobility is introduced, higher connectivity levels are achieved with the same number of DM nodes. This shows that nodes with multiple transmission capability can be used to improve the performance in dynamic scenarios.

Table III - Mobility Model Parameters

Parameters	Value
Maximal Speed (v)	10 m/s
Pause Time (T_p)	20 s, 5 s
Direction of movement	Uniformly dist. $[0, 2\pi)$
Broadcast period (T_b)	50 s, 25 s

In applications such as sensor networks, where mobility capabilities are fairly restricted, the DM nodes can be introduced as special mobile base stations to monitor the area and also to ensure an efficient operation of the network. To simulate this application scenario, we also considered the case where only DM nodes are mobile. Figure 7(b) shows the results for $r_f=3r$ and $r_f=5r$ in a static network and when only the DM nodes are mobile. Only a slight improvement is achieved with mobile DM nodes for both values of r_f , as compared to the static case. A high fraction f (at least 50% of DM nodes) still required to obtain highly connected topologies. Thus, in both cases (static and mobile) the effect of the DM nodes is limited by the small density of nodes in the area.

Figure 8(a) shows the results of similar experiments increasing the nodes density (250 nodes in a 1kmx1km square). As in the static scenario, a transition from a disconnected to a connected network also occurs under mobility as f increases, for both values of r_f and the mobility effect can also be seen in Figure 8(a). We can also note that, for high connected topologies in the mobile case ($f>0.1$), no significant difference is observed by increasing r_f from $3r$ to $5r$. This fact is important because it limits the transmitting power requirements of the DM nodes to achieve high connectivity.

Finally, we performed simulations with different pause time values in order to understand how the mobility level of the DM nodes impacts the connectivity. As discussed earlier, the broadcast period (T_b) is also important when we take mobility into account. Figure 8(b) shows the results for a 250 nodes network. We used $T_p=20$ s and $T_p=5$ s to simulate low and high mobility levels, respectively, and we also used two different broadcast periods, namely $T_b=50$ s and $T_b=25$ s. High connectivity levels with very low variability were obtained with f as low as 0.1 in the mobile scenario and no significant difference is observed for different combinations of T_p and T_b . On the other hand, for disconnected topologies ($f<0.1$), no clear relation between pause time, broadcast period and connectivity could be observed. In fact, a high variability was observed for lower connectivity levels for all scenarios.

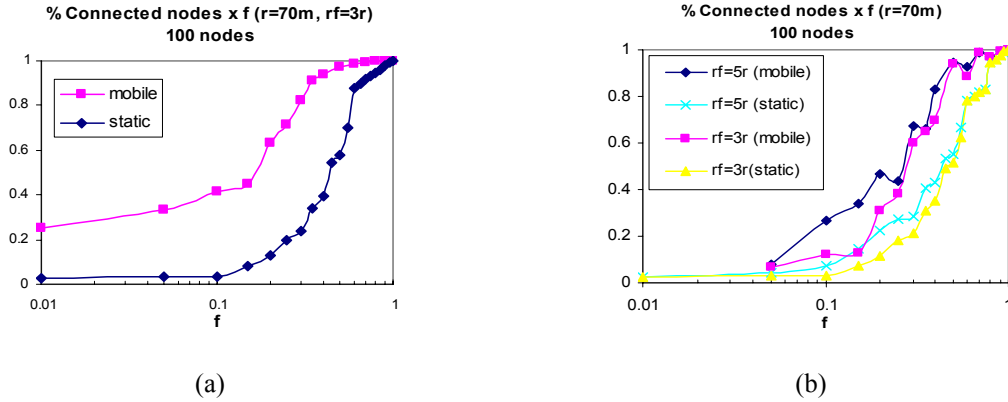


Figure 7: (a) Connectivity vs. f for static and mobile scenarios. (b) Connectivity vs. f for different r_i , with only DM node mobile ($T_p = 20$ s and $T_b = 50$ s).

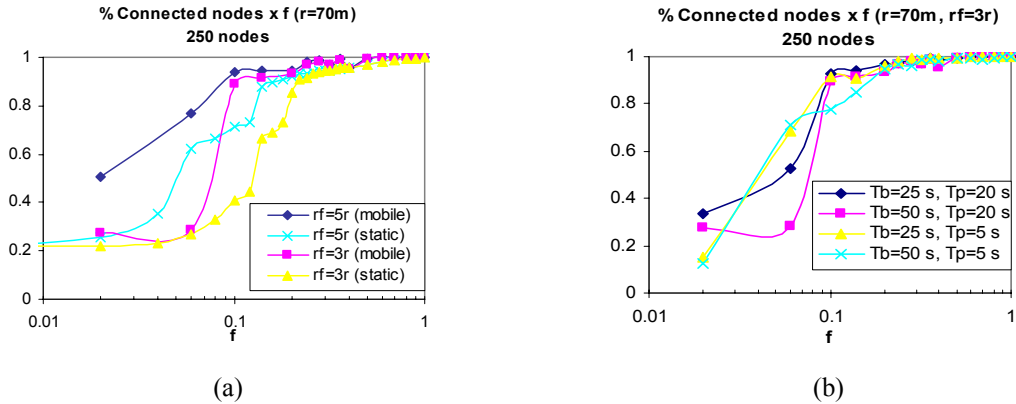


Figure 8: (a) Connectivity vs f in static and mobile scenario with $T_p = 20$ s and $T_b = 50$ s. (b) Connectivity vs. f with $r_i=3r$ for different combinations of T_b and T_p .

6. Conclusion

Connectivity directly affects the efficiency of multihop routing, and consequently the overall performance of wireless ad hoc networks. Most existing research on connectivity in ad hoc or sensor networks consider homogeneous networks. The development of a more robust analytical framework for analyzing a network with multi-radio nodes is an important problem, since the availability of multiple interfaces in the same host is expected to be common in the future. In this paper, we have analyzed the connectivity in a heterogeneous ad hoc network with Dual-Mode nodes. We developed an analytical model for the one-dimensional case and presented simulation results for the two dimensional scenario showing that a small number of DM nodes can significantly increase the network connectivity in both static and mobile networks. Our work can be extended to include nodes with more than two communication radii. As future work, we will consider an analytical framework for the two dimensional case in order to obtain bounds on the parameters required to ensure high

connectivity in heterogeneous networks. Other issues related to network capacity and multi-radio aware routing need also to be further investigated in heterogeneous networks.

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